

System Developments for High-Precision Multisensor Navigation for Autonomous Driving and Flying

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RaD

www.dfhbf.de, www.goca.info, www.moldpos.eu
www.navka.de



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Study Programmes of the Faculty IMM with Contents related to Geodesy/Geomatics and Navigation

(www.hs-karlsruhe.de/)

Research Oriented (BSc – MSc – PhD)

Hochschule Karlsruhe Technik und Wirtschaft
UNIVERSITY OF APPLIED SCIENCES
Näher dran.

Bachelor
Geodäsie und Navigation
Bachelor of Science (B.Sc.)
Fakultät für Informationsmanagement und Medien

Hochschule Karlsruhe Technik und Wirtschaft
UNIVERSITY OF APPLIED SCIENCES
Näher dran.

Master
Geomatics
International Degree Program /
Consecutive Degree Program
Master of Science (M.Sc.)
Faculty of Information Management and Media (IMM)

Hochschule Karlsruhe Technik und Wirtschaft
UNIVERSITY OF APPLIED SCIENCES
Näher dran.

Bachelor
Verkehrssystemmanagement
Bachelor of Engineering (B.Eng.)
Fakultät für Informationsmanagement und Medien
Neu ab Wintersemester 2012/13

Hochschule Karlsruhe Technik und Wirtschaft
UNIVERSITY OF APPLIED SCIENCES
Näher dran.

Bachelor
Geoinformationsmanagement
mit den Vertiefungsrichtungen:
Geomarketing, Kartographie und Geomedien, Umwelt
Bachelor of Science (B.Sc.)
Fakultät für Informationsmanagement und Medien



RaD Project

www.navka.de



GNSS / MEMS / MOEMS Algorithms and Software for Out-/Indoor Navigation (People, Vehicles, Goods) and Georeferencing with distributed Sensors and Platforms

$$y = \begin{bmatrix} x^e & y^e \end{bmatrix}$$

$$\begin{bmatrix} b, z & | & s \end{bmatrix}^T$$

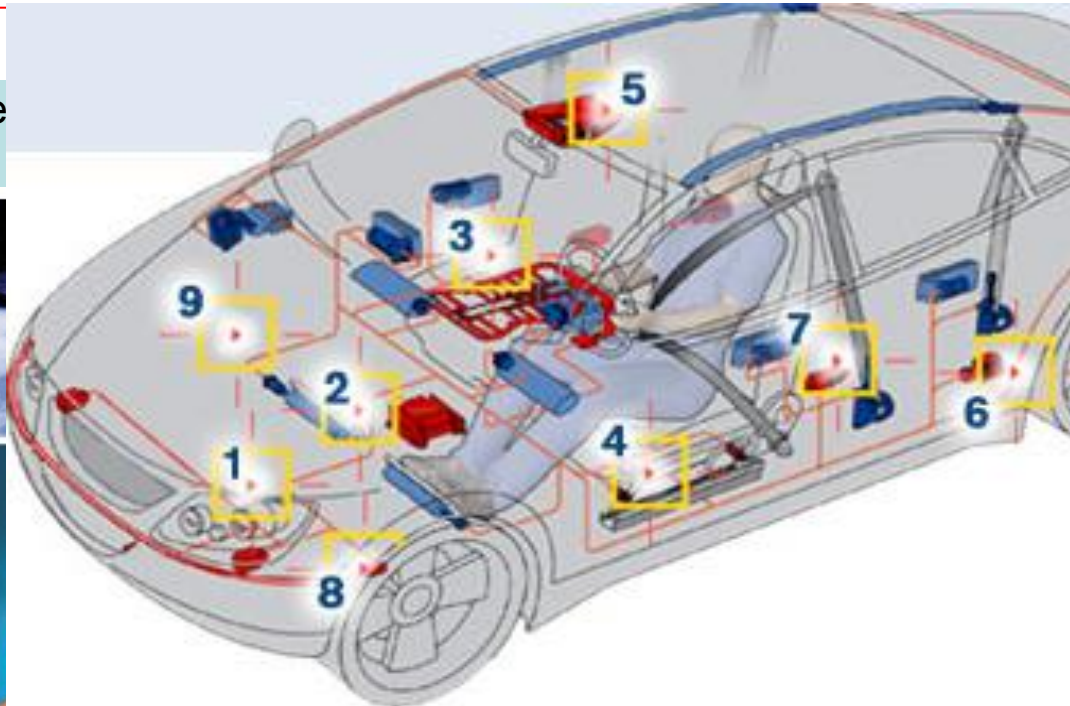
GNSS

Deep Coupling

- Code
- Phase
- Doppler

Tight Coupling

- Position
- Velocity



MEMS MOEMS

Deep Coupling

- Gyroscopes
- Accelerometers
- Magnetometers
- Camera
- Barometer



<https://www.youtube.com/watch?v=-k--3GxrQXU>

Autonomous Driving (... and Flying)

Level 0

- ▶ Gesamte Kontrolle liegt ganzzeitig beim Fahrer

Level 1

- ▶ Unterstützung spezieller Funktionen
- ▶ ESP, ABS

Level 2

- ▶ Zwei oder mehr Funktionen werden unterstützt und können miteinander interagieren
- ▶ Tempomat in Verbindung mit Fahrbahnassistentz, Parkhilfe

Level 3

- ▶ Gesamte Kontrolle kann vom Fahrer abgegeben werden, der Fahrer muss jedoch jederzeit einsatzbereit sein und im Zweifelsfall manuell eingreifen können

Level 4

- ▶ Gesamte Kontrolle liegt ganzzeitig beim Fahrzeug, das Fahrzeug fährt völlig unabhängig

$$e(t) = y(t)_{\text{Desired State}} - y(t)_{\text{Navigated State}}$$

Main Topics

$e(t)$ = Control Deviation (Regelabweichung)

$u'(t)$ = Control Variables, e.g. Thrust & Torque

$u(t)$ = Control, e.g. Propeller-Rotations

SLAM

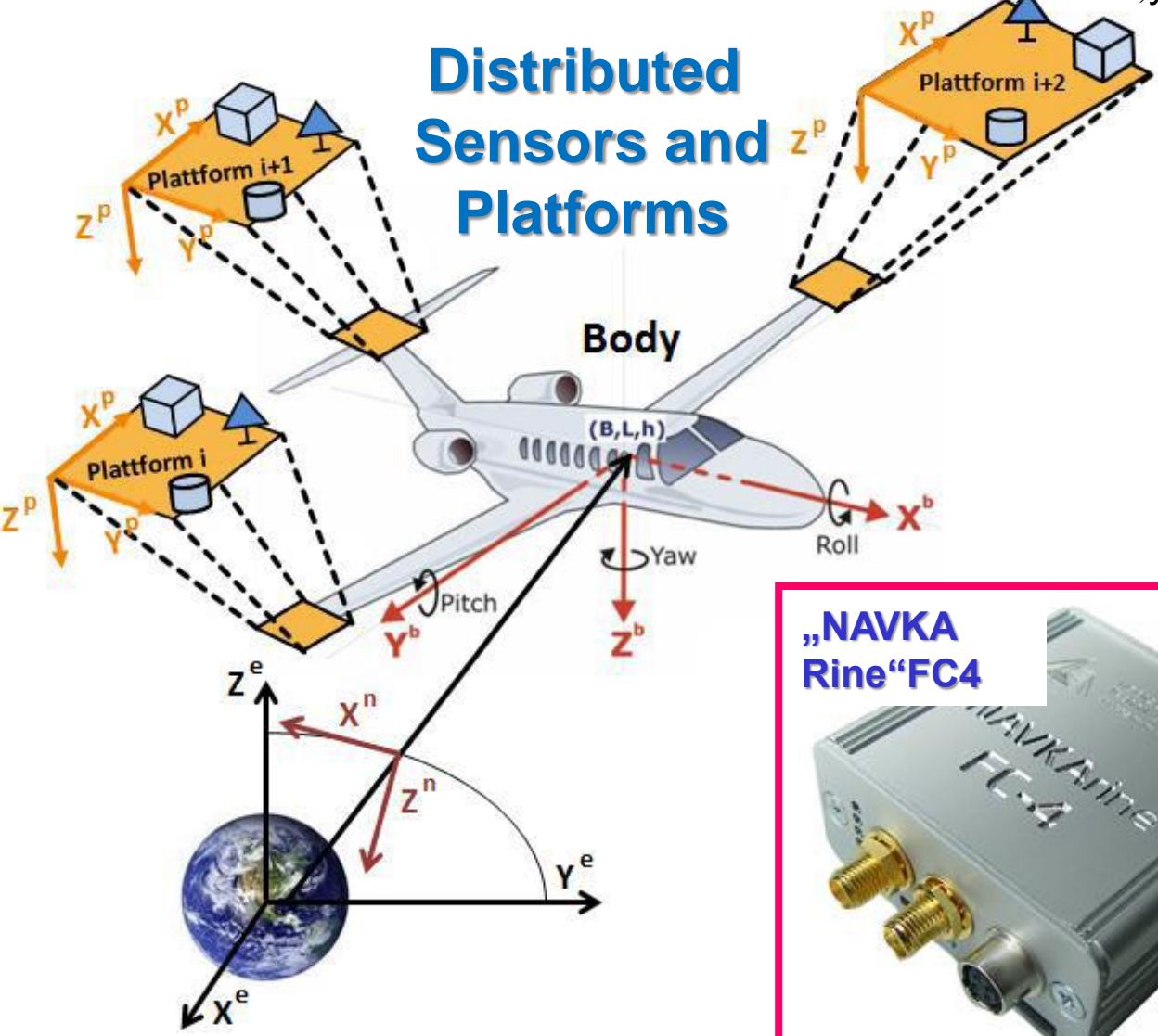
- Obstacle Detection
- Collision Avoidance
- Dynamic Path Planning



Navigation with Distributed Sensors & Platforms (NAVKA-Concept)

$$y = \left[x^e \ y^e \ z^e \mid \dot{x}^e \ \dot{y}^e \ \dot{z}^e \mid r^e \ p^e \ y^e \mid \ddot{x}^e \ \ddot{y}^e \ \ddot{z}^e \mid \omega_{eb,x}^b \ \omega_{eb,y}^b \ \omega_{eb,z}^b \mid \mathbf{s} \right]^T$$

Distributed Sensors and Platforms



MaxPlanck
München

Navigation Frames



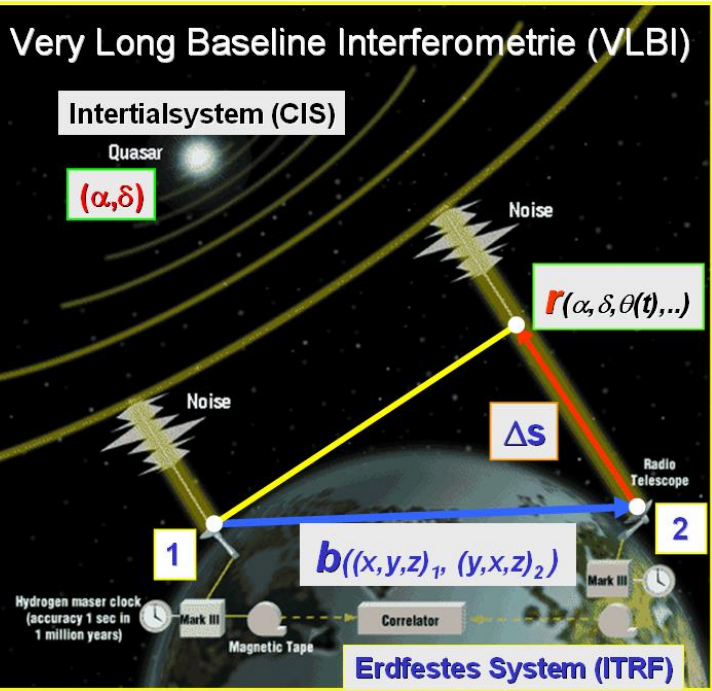
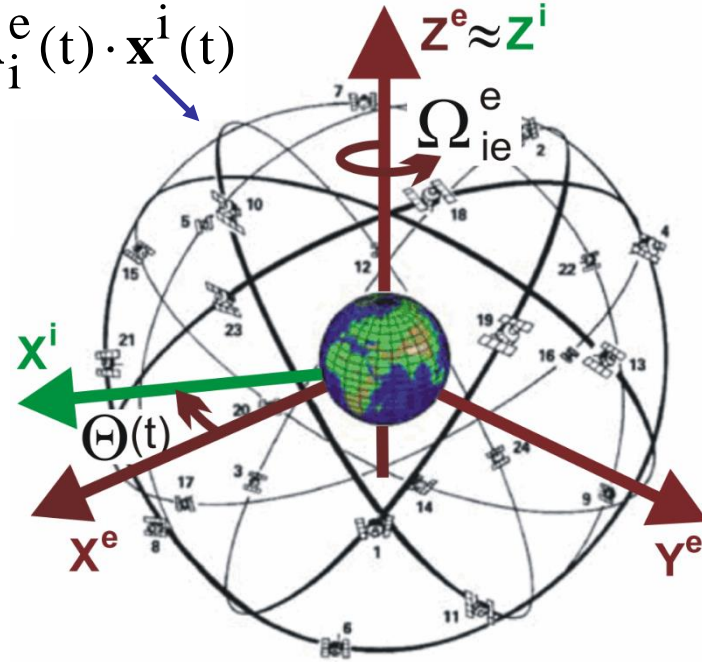
Inertial Frame „(i)-Frame“ – 600 Quasars (α, δ)

$$\mathbf{R}_i^e(t) = (\mathbf{R}_e^i(t))^T = \mathbf{R}_P \cdot \mathbf{R}_E \cdot \mathbf{R}_N \cdot \mathbf{R}_{Pr}$$

$$\mathbf{x}^e(t) = \mathbf{R}_i^e(t) \cdot \mathbf{x}^i(t)$$

$$\mathbf{r}^e(t) = \mathbf{R}_i^e(t) \cdot \mathbf{r}^i(t)$$

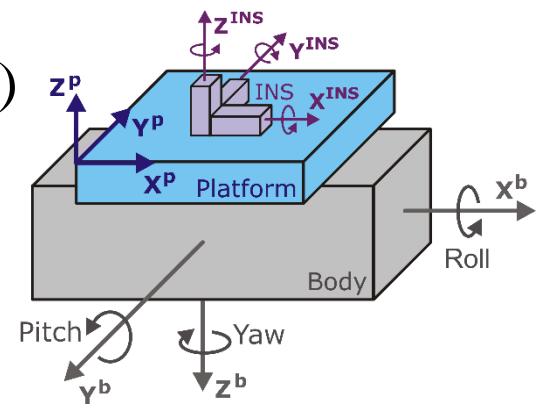
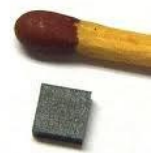
$$\mathbf{r}^i = \begin{bmatrix} \cos \delta \cdot \cos \alpha \\ \cos \delta \cdot \sin \alpha \\ \sin \delta \end{bmatrix}$$



$$\ddot{\mathbf{x}}^i(t) = \mathbf{g}^i(\mathbf{x}) + \mathbf{a}^i(\text{Sensor}, t) = \mathbf{g}^i(\mathbf{x}) + \mathbf{R}_b^i(t) \cdot \mathbf{a}^b(\text{Sensor}, t)$$

$$\mathbf{\Omega}_{ib}^b(\text{Sensor})$$

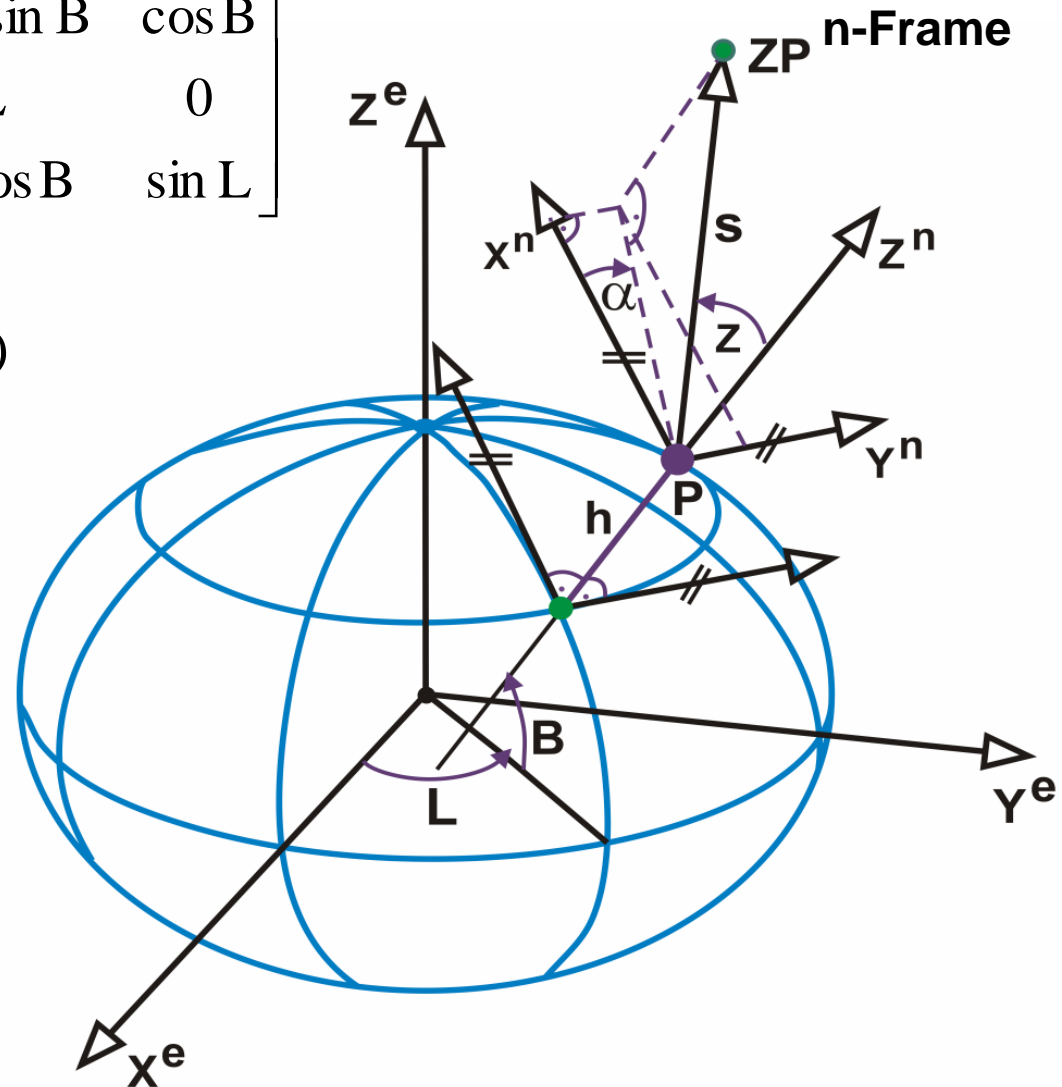
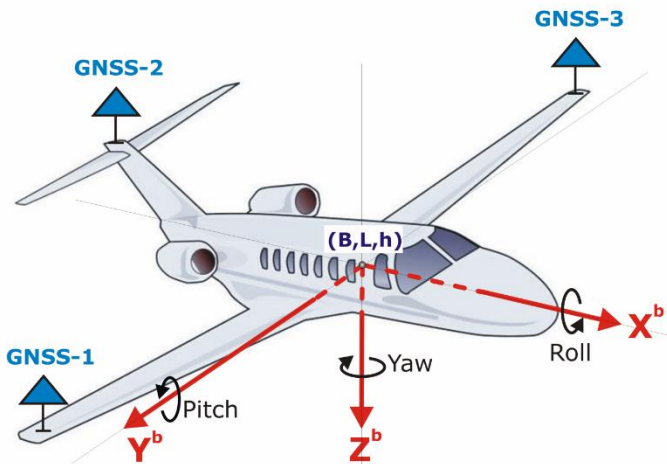
$$\dot{\mathbf{R}}_b^i(t) = \mathbf{R}_b^i(t) \cdot \mathbf{\Omega}_{ib}^b(t)$$



Navigationframe - „Navigation-Frame“ oder „n-Frame“

$$\mathbf{R}_e^n(B, L) = \begin{bmatrix} -\cos B \cdot \sin L & -\sin L \cdot \sin B & \cos B \\ -\sin L & \cos L & 0 \\ \cos L \cdot \cos B & \sin L \cdot \cos B & \sin L \end{bmatrix}$$

$$\mathbf{x}^{n,i} = \mathbf{R}_e^n(B, L) \cdot (\mathbf{x}^{e,i} - \mathbf{x}(B, L, h))^e$$

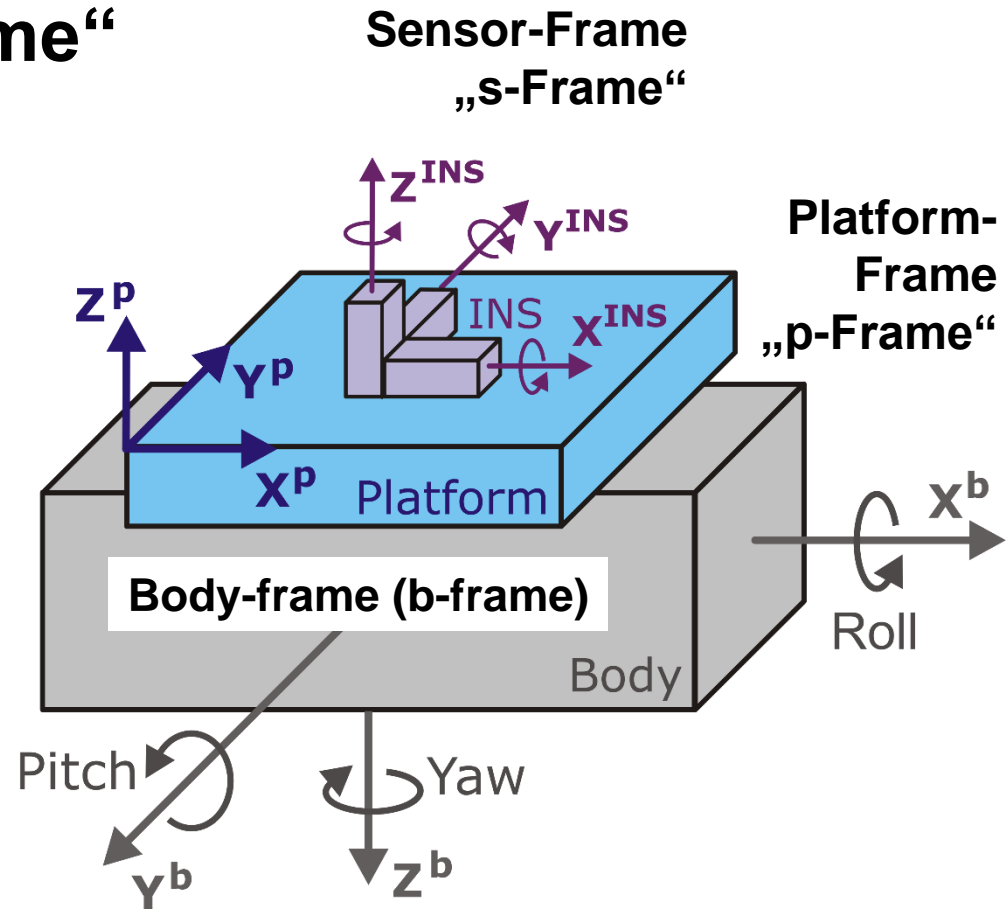


Body-Frame - „b-Frame“

$$\mathbf{x}^{b,i} = \mathbf{R}_n^b(r, p, y) \cdot \mathbf{x}^{n,i}$$

$$\begin{bmatrix} r \\ p \\ y \end{bmatrix} = \begin{bmatrix} \tan^{-1}[\mathbf{R}_b^n(3,2) / \mathbf{R}_b^n(3,3)] \\ \tan^{-1}[-\mathbf{R}_b^n(3,1) / \sqrt{\mathbf{R}_b^n(2,1)^2 + \mathbf{R}_b^n(1,1)^2}] \\ \tan^{-1}[\mathbf{R}_b^n(2,1) / \mathbf{R}_b^n(1,1)] \end{bmatrix}$$

$$\mathbf{R}_n^b = \begin{pmatrix} \cos p \cos y & \cos p \sin y & -\sin p \\ \sin r \sin p \cos y - \cos r \sin y & \sin r \sin p \sin y + \cos r \cos y & \sin r \cos p \\ \cos r \sin p \cos y + \sin r \sin y & \cos r \sin p \sin y - \sin r \cos y & \cos r \cos p \end{pmatrix}$$



GNSS and Challenges



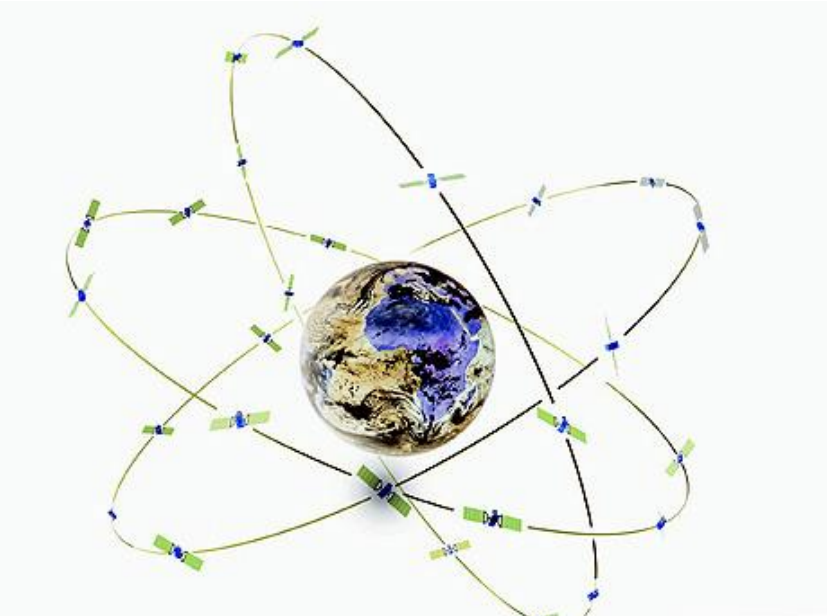
Orbit Design of the 4 GNSS (without Augmentation Systems)



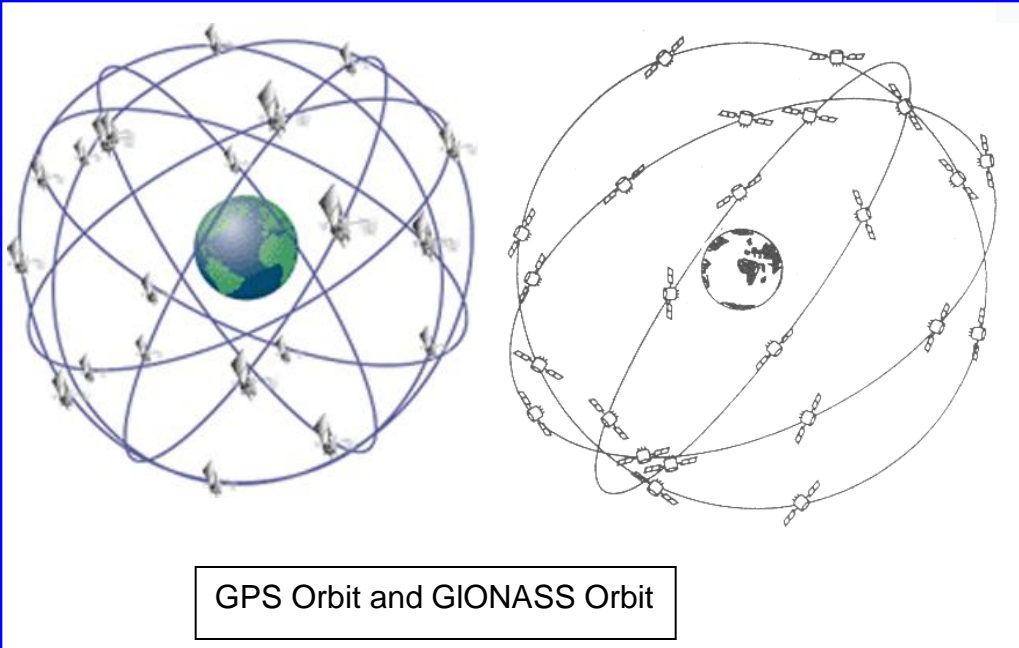
28 Dec.2005 **Giove-A** lift off
05:19 UTC Baikonour,
Kazakhstan



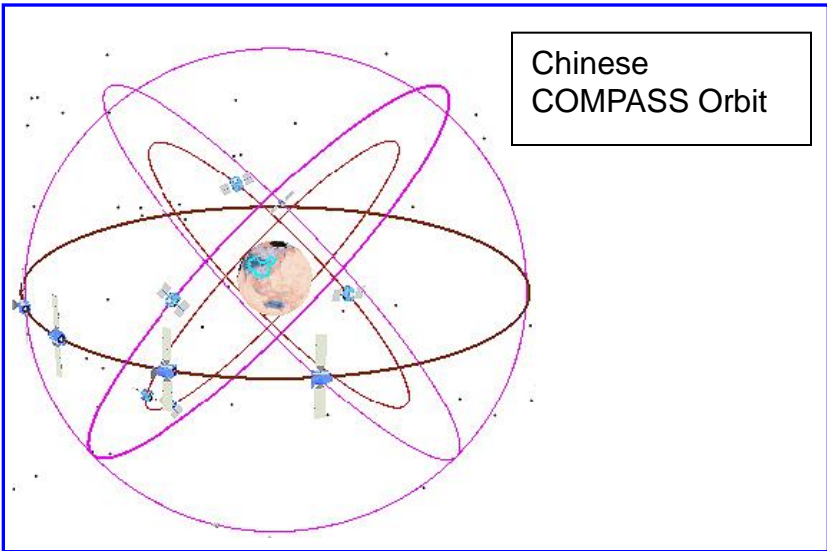
Galileo Satellite



Orbit – Segment Galileo



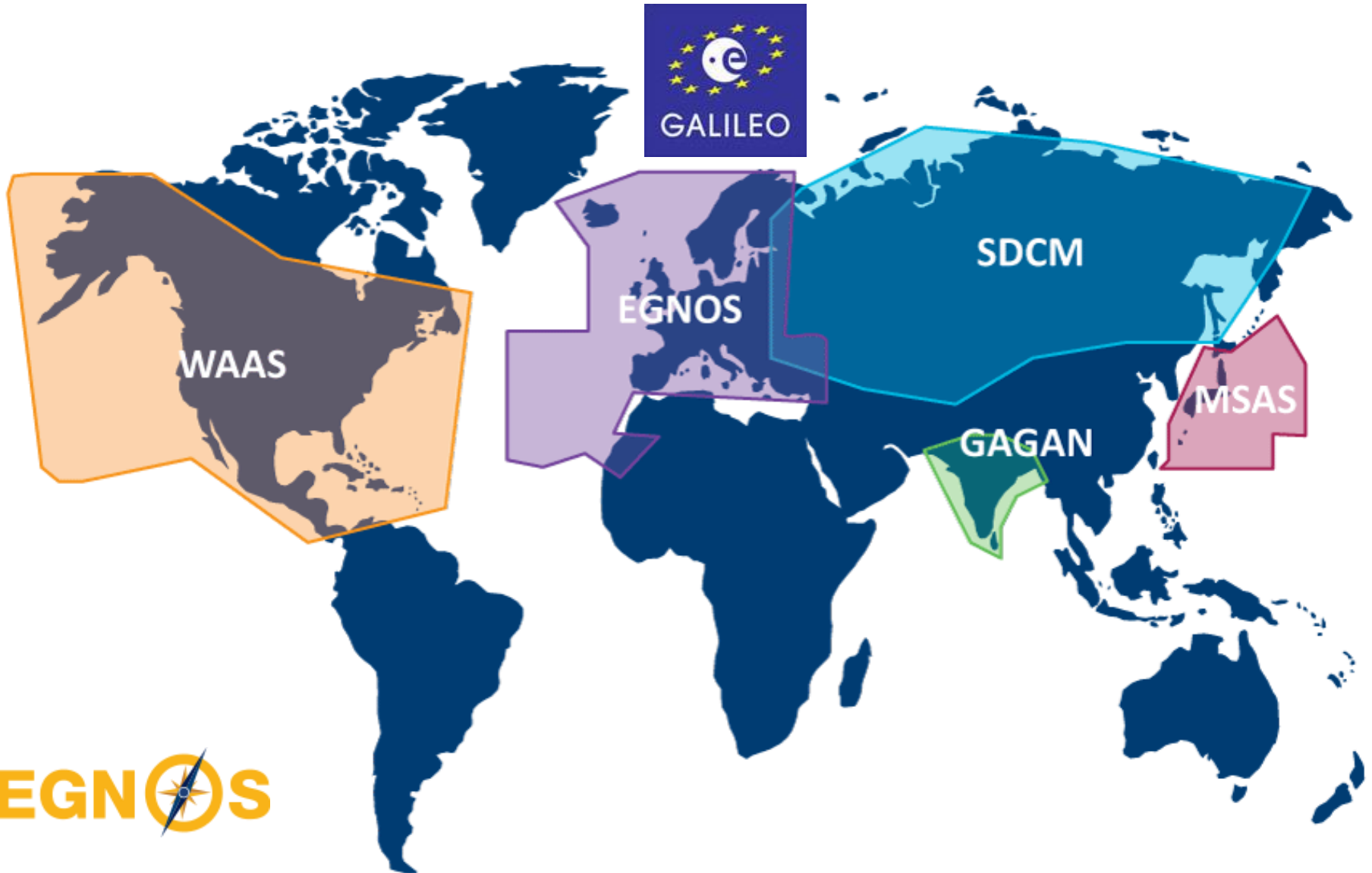
GPS Orbit and GIONASS Orbit



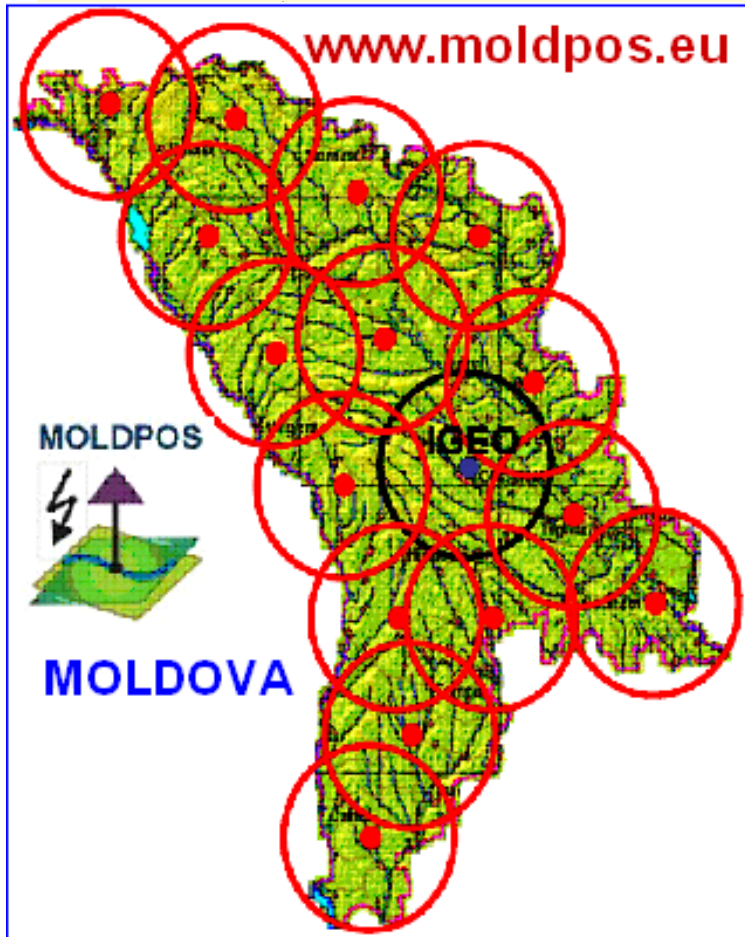
Chinese
COMPASS Orbit

Space/Satellite Based Augmentation Systems (SBAS)

..... für DGNSS-Codemessungen (DGNSS-Korrekturen. Standard RTCM oder RTCA)

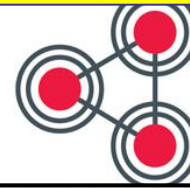


Regional Precise GNSS-Positioning Services – Worldwide Frame in Europe: ETRF89 („Frozen Plate“). same as EUREF-IP Service

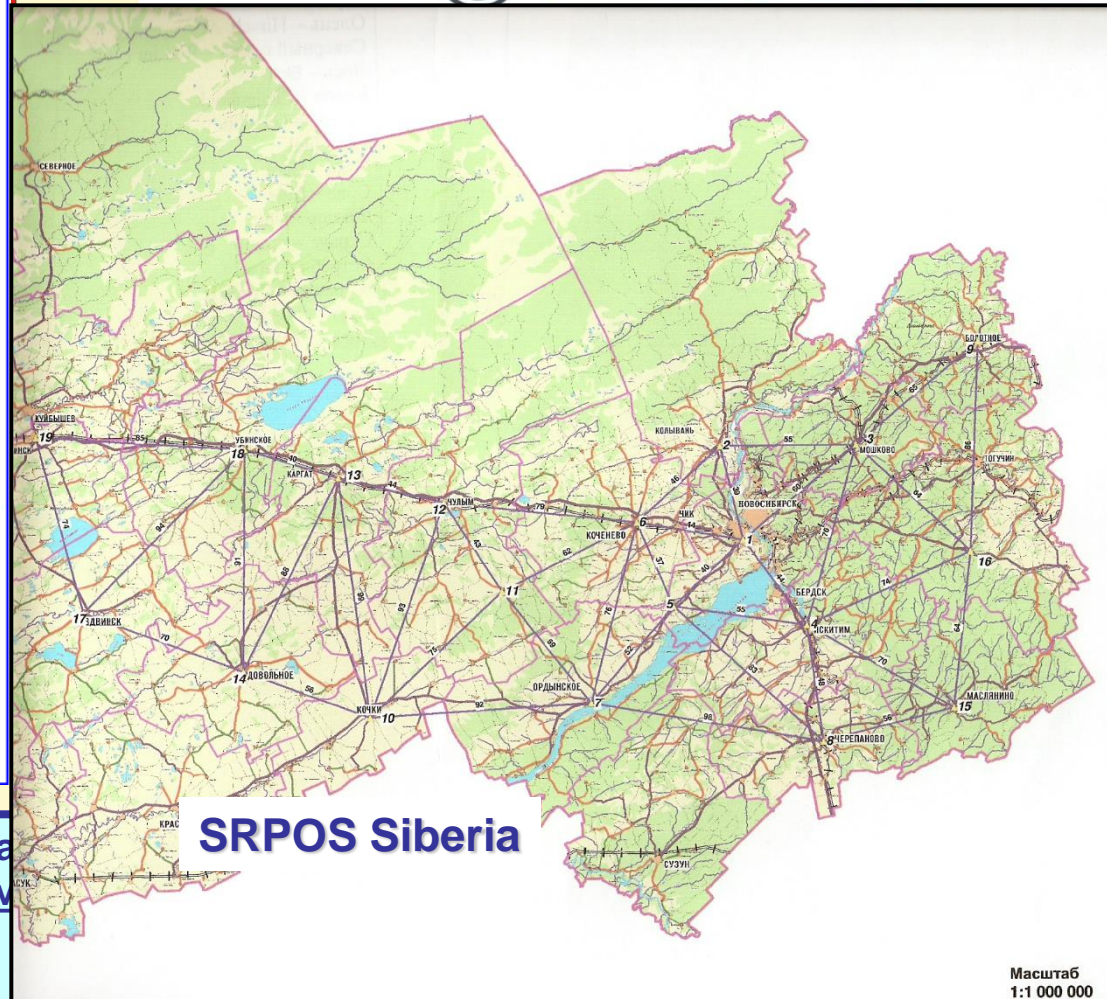


1

2



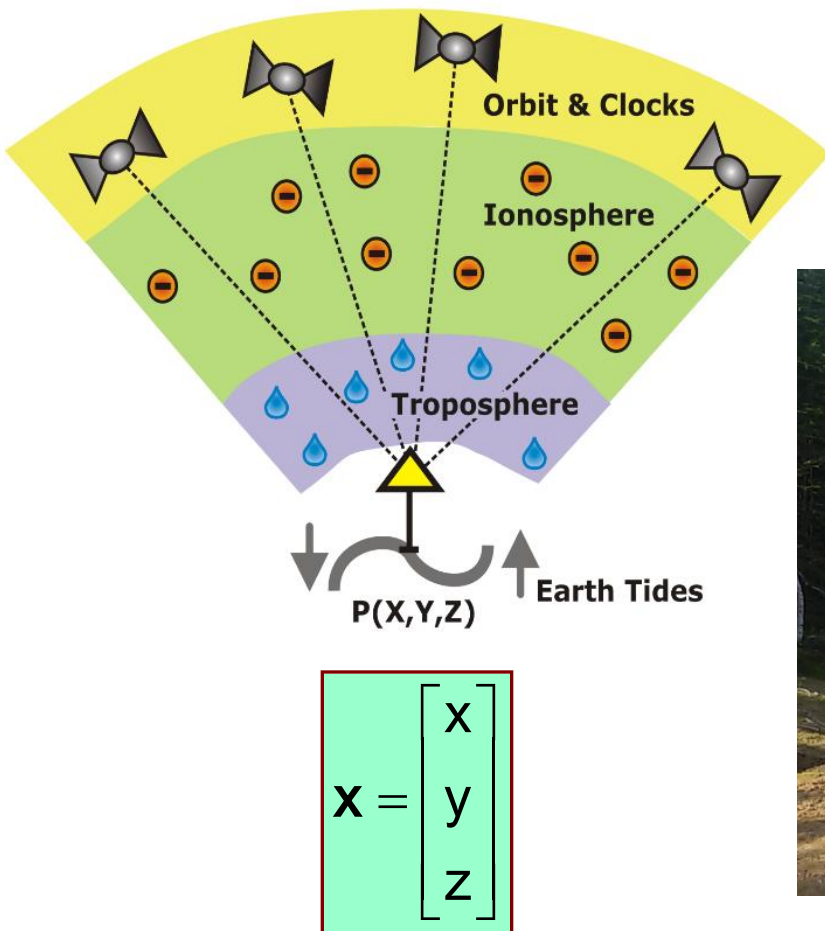
SmartNet
 **EUROPE**



4 RTCM Services: 1-3 cm Horizontal and Vertical Accuracy
 Geodetic Infrastructures for GNSS-Service

www.moldpos.eu
www.geozilla.de

GNSS-Positions – necessary to be integrated in „OPKP age“ and „GNSS/MEMS sensor fusion age“.

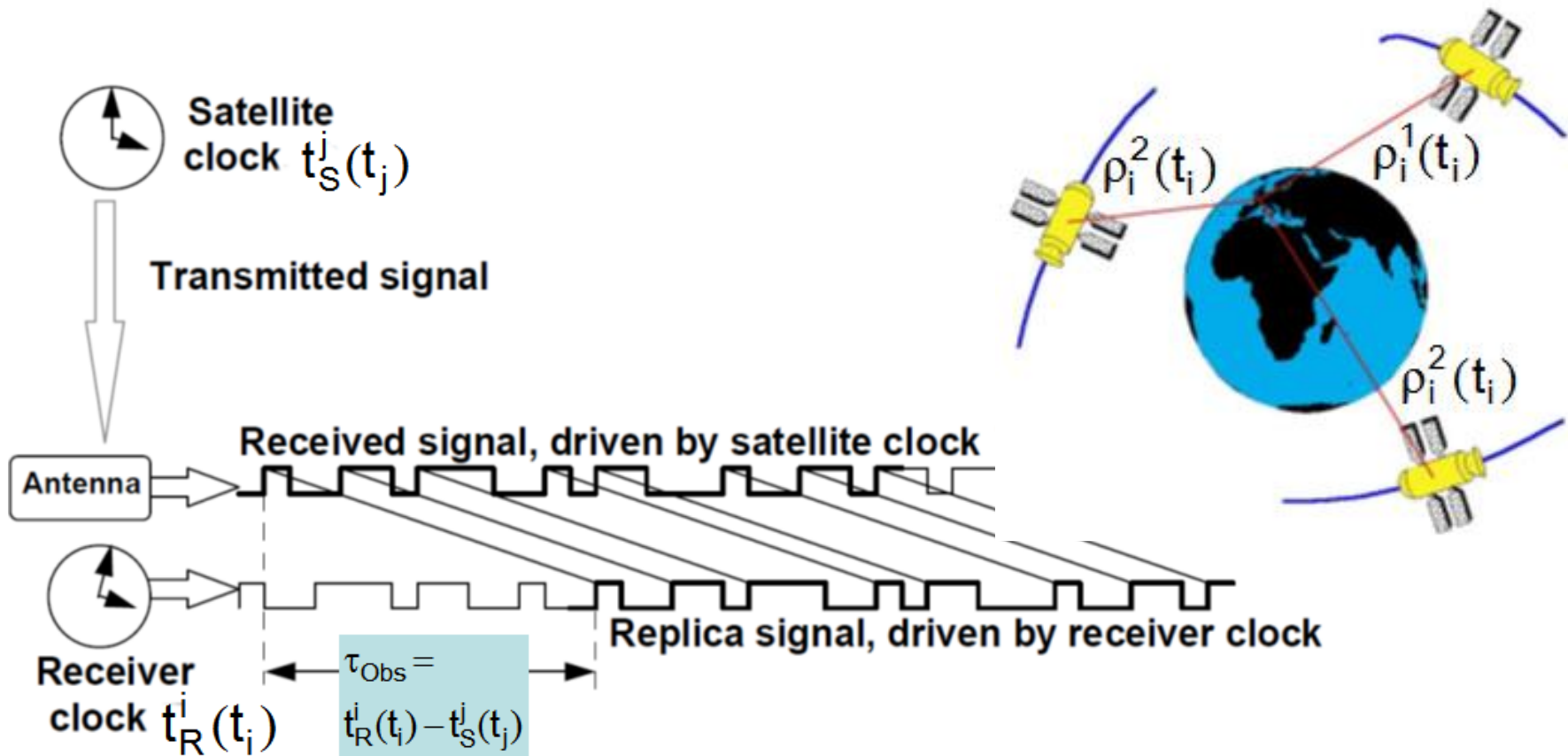


Useful for extended monitoring scenarios, e.g. large pipeline networks



Pseudorange or Code-Observation

$$\rho_i^j(t_i)_{\text{Obs}} = c \cdot \tau_{i\text{Obs}}^j = c \cdot (t_R^i(t_i) - t_S^j(t_j)) = c \cdot ((t_i + \Delta\bar{t}_{R,i}) - (t_j + \Delta\bar{t}_{S,j}))$$



Pseudorange Modeling in ECEF and GNSS-time

$$\rho_i^j(t_i)_{\text{Obs}} = c \cdot \tau_{i\text{Obs}}^j = c \cdot (t_R^i(t_i) - t_S^j(t_j)) = c \cdot ((t_i + \Delta\bar{t}_{R,i}) - (t_j + \Delta\bar{t}_{S,j}))$$

$$\text{with } \tau_{i\text{Obs}}^j = t_R^i(t_i) - t_S^j(t_j)$$

1. Without Atmosphere corrections

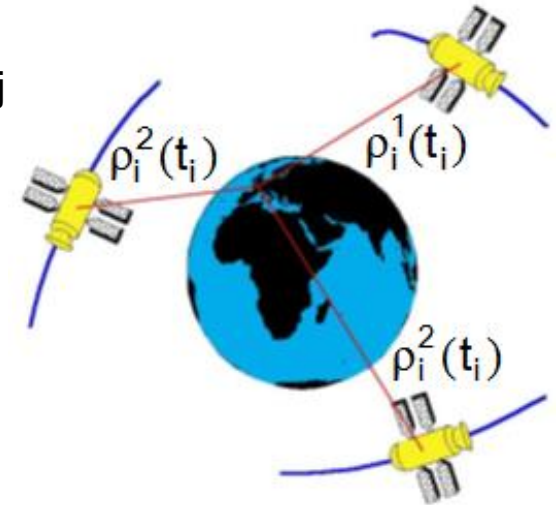
$$\rho(t_i)_{\text{Obs}} = c \cdot (t_R^i(t_i) - t_S^j(t_j)) = c \cdot (t_i - t_j) + c \cdot \Delta\bar{t}_{R,i} - c \cdot \Delta\bar{t}_{S,j}$$

$$\rho(t_i)_{\text{Obs}} = \tilde{\rho}(t_i, t_j, \mathbf{o}^j) + c \cdot \Delta\bar{t}_{R,i} - c \cdot \Delta\bar{t}_{S,j}$$

$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R^i(t_i) - \tilde{\mathbf{x}}_S^j(t_j, \mathbf{o}^j) \right| + c \cdot \Delta\bar{t}_{R,i} - c \cdot \Delta\bar{t}_{S,j}$$

$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R^i(t_i) - \tilde{\mathbf{x}}_S^j(t_i - \tau_i, \mathbf{o}^j) \right| + c \cdot \Delta\bar{t}_{R,i} - c \cdot \Delta\bar{t}_{S,j}$$

$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R^i(t_i) - \tilde{\mathbf{x}}_S^j\left(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j\right) \right| + c \cdot \Delta\bar{t}_{R,i} - c \cdot \Delta\bar{t}_{S,j} \quad \text{with} \quad \tilde{\rho}(t_i) = \left| \tilde{\mathbf{x}}_R^i(t_i) - \tilde{\mathbf{x}}_S^j\left(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}\right) \right|$$

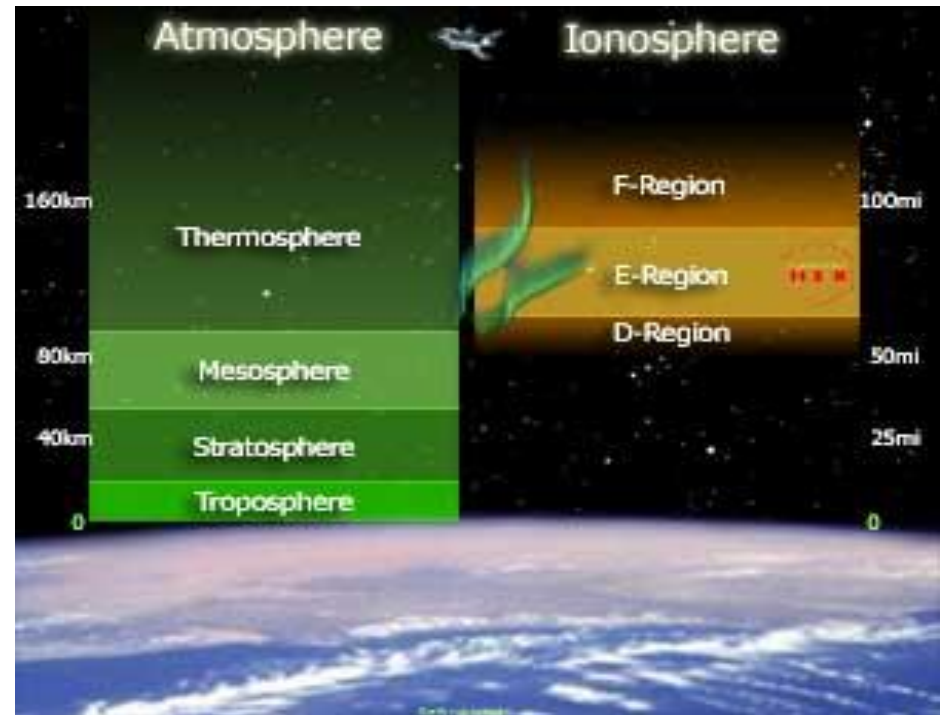
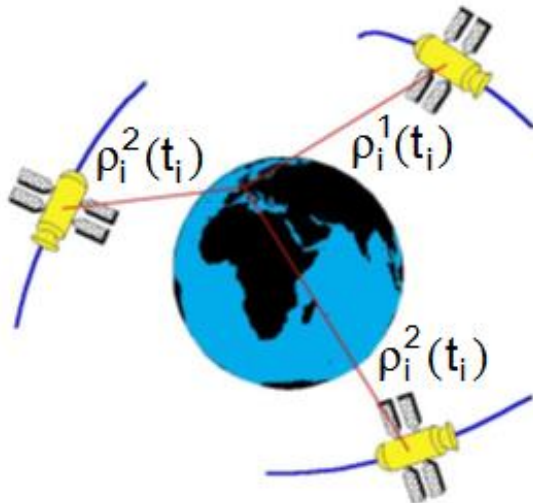


Pseudorange Modeling in ECEF and GNSS-time

2. With Atmosphere (Ionosphere and Troposphere)

$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}), \mathbf{o}^j \right| + c \cdot \Delta \bar{t}_{R,i} - c \cdot \Delta \bar{t}_{S,j} + \Delta \rho_{i,\text{ION}}^j + \Delta \rho_{i,\text{TROP}}^j$$

$$\text{with } \tilde{\rho}_i^j(t_i) = \left| \tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}) \right|$$



Light Time Equation

$$\left| \underbrace{\tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j)}_{\tilde{\rho}_i^j(t_i) \text{ - Iteration: "Light Time Equation"}} \right|$$

- Can be solved iteratively exactly based on GNSS time $t(i)$, Orbit \mathbf{o}^j and good position $\mathbf{x}_R(t(i))$
- Must be solved in repeated parameter estimation, if $\Delta t_{R,i}(t_i)$ and $\mathbf{x}_R(t(i))$ are unknown

3. Geodynamic Corrections

1. 3D-Earth Tide Corrections

$$\Delta r = \sum_{j=2}^3 \frac{GM_j}{GM} \frac{r^4}{R_j^3} \left\{ [3l_2(\mathbf{R}_j * \mathbf{r})\mathbf{R}_j] + \left[3\left(\frac{h_2}{2} - l_2\right)(\mathbf{R}_j * \mathbf{r})^2 - \frac{h_2}{2} \right] \mathbf{r} \right\} [-0.025 \sin \rho \cos \rho \sin(\theta_g + \lambda)] \mathbf{r}$$

3.2. Ocean Loading (IERS Standards, 1996)

3.3. Atmospheric Loading

3.4. Earth Orientation

$$\mathbf{R}_i^e(t) = \mathbf{R}_P \cdot \mathbf{R}_E \cdot \mathbf{R}_N \cdot \mathbf{R}_{Pr} \quad \mathbf{x}^e(t) = \mathbf{R}_i^e(t) \cdot \mathbf{x}^i(t)$$

Pseudorange Modeling in ECEF and GNSS-time

$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S\left(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j\right) \right| + c \cdot \Delta \bar{t}_{R,i} - c \cdot \Delta \bar{t}_{S,j} + \Delta \rho_{i,\text{ION}}^j + \Delta \rho_{i,\text{TROP}}^j$$

$$\text{with } \rho(t_i)_{\text{Obs}} = \left| \underbrace{\tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S\left(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j\right)}_{\tilde{\rho}_i^j(t_i) \text{ - Iteration: "Light Time Equation"}} \right| \quad \text{and} \quad \tau_i = \frac{\tilde{\rho}_i^j(t_i)}{c}$$

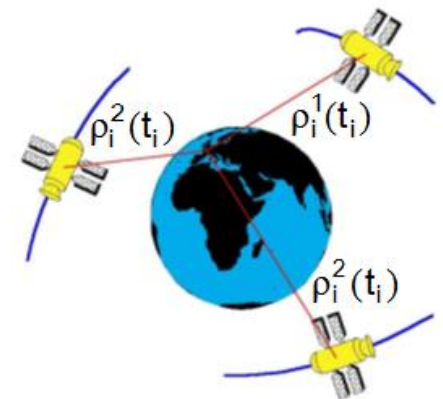
4. Additionally: Relativistic effects (Special and General Relativity)

4.1 Special and General Relativity in general

$$\frac{t'-t}{t} = \frac{\Delta t}{t} = \frac{x'-x}{x} = \frac{\Delta x}{x} = -\frac{f'-f}{f} = -\frac{\Delta f}{f} = \frac{m'-m}{m} = \frac{\Delta m}{m} = -\left(\frac{1}{2} \cdot \left(\frac{v}{c}\right)^2 + \frac{\Delta U}{c^2}\right)$$

4.2 GNSS pseudorange-observation scenario

$$"t" = \tau_i = \frac{\tilde{\rho}_i^j(t_i)}{c} \quad \text{and} \quad \frac{\Delta U}{c^2} = \frac{\mu}{c^2} \cdot \left[\frac{1}{R_E + h(t_i)} - \frac{1}{R_E} \right]$$



Pseudorange Modeling in ECEF and GNSS-time

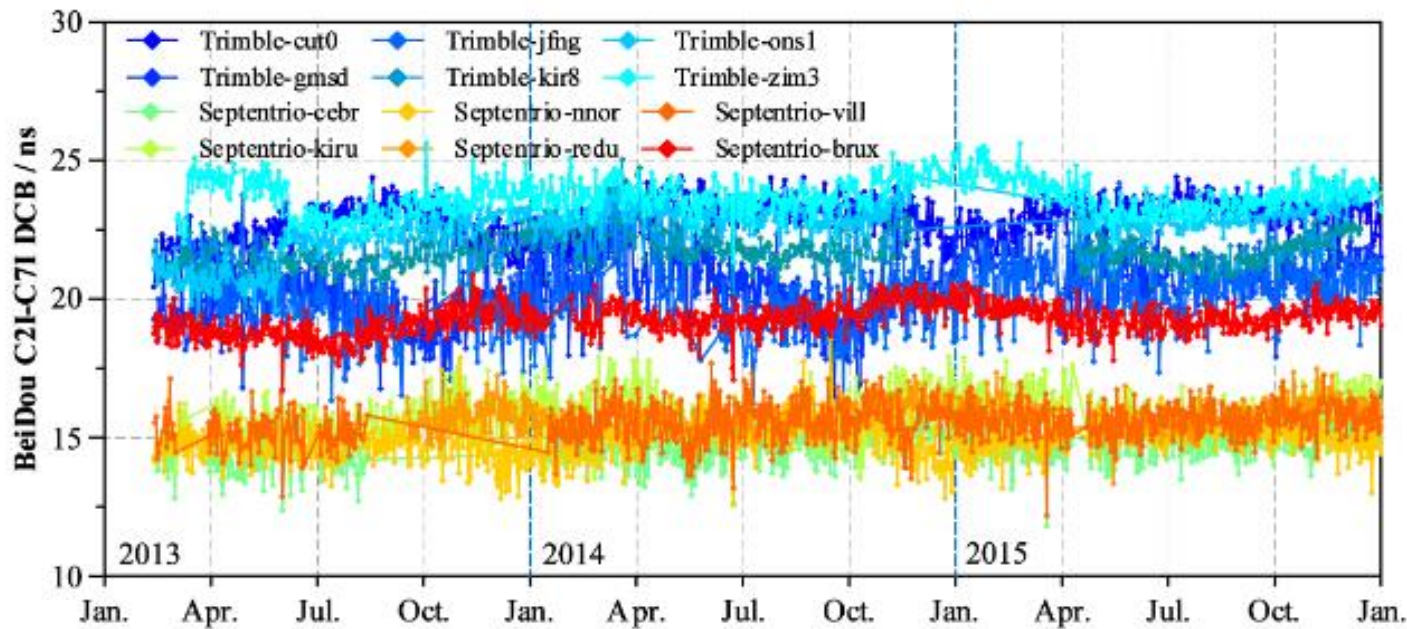
$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S\left(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j\right) \right| + c \cdot \Delta \bar{t}_{R,i} - c \cdot \Delta \bar{t}_{S,j} + \Delta \rho_{i,\text{ION}}^j + \Delta \rho_{i,\text{TROP}}^j$$

Further Clock/Time-Bias

Trimble NETR9 receivers: 18.0~24.0 ns

Septentrio receivers: 13.0~19.0 ns

- **GNSS differential code biases (DCB)**



Time series of BeiDou C2I-C7I DCBs for the selected receivers during the period 2013–2015

DCB products available from 01/2013
 IGG – updated daily (daily interval)
 DLR – updated quarterly (both weekly and daily intervals)

Pseudorange Modeling in ECEF and GNSS-time

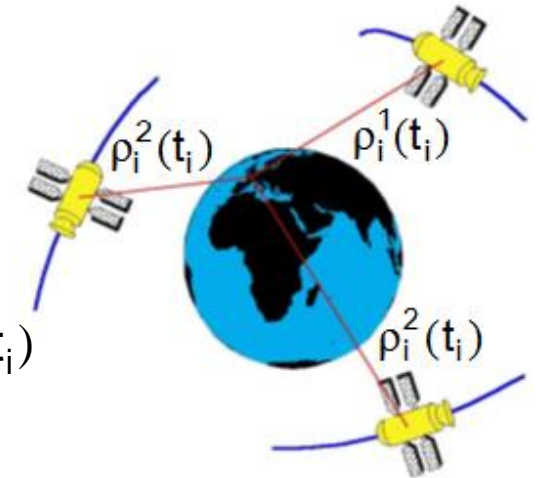
$$\rho(t_i)_{\text{Obs}} = \left| \tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j) \right| + c \cdot \Delta t_{R,i} - c \cdot \Delta t_{S,j} + \Delta \rho_{i,\text{ION}}^j + \Delta \rho_{i,\text{TROP}}^j$$

5. Final Rough Pseudo-Range observation equation

$$\rho(t_i)_{\text{Obs}} = \rho(t_i)_{\text{Obs}} = \left| \underbrace{\tilde{\mathbf{x}}_R(t_i) - \tilde{\mathbf{x}}_S(t_i - \frac{\tilde{\rho}_i^j(t_i)}{c}, \mathbf{o}^j)}_{\tilde{\rho}_i^j(t_i) \text{ - Iteration: "Light Time Equation"}} \right| + c \cdot (\Delta t_{R,i}(t_i) + \frac{1}{c^2} \cdot (\mathbf{x}_R^i(t_i) - \mathbf{x}_S^j(t_i)) \cdot (\boldsymbol{\omega}_E \times \mathbf{x}_R^i(t_i)) - c \cdot (\Delta t_{S,j}(t_i) - \left[\frac{2}{c^2} \sqrt{\mu \cdot a} \cdot e \cdot \sin(E(t_i)) \right]^j) + \Delta \rho_{i,\text{ION}}^j(t_i) + \Delta \rho_{i,\text{TROP}}^j(t_i)$$

Observations:

$$t_R^i(t_i) \text{ and } \rho_i^j(t_i)_{\text{Obs}} = c \cdot \tau_{i,\text{Obs}}^j$$



Unknowns: $(x, y, z)_R(t_i), \Delta t_{R,i}(t_i)$

Corrections: Sagnac R_i , Relativity S_j , Clock S_j

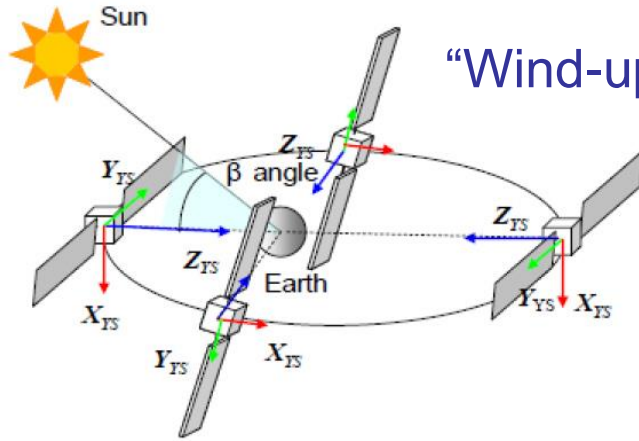
Further Corrections, Add. rel. clock variations, Add. ModelsION, TROP, Earth Dynamics

Phase Modelling in OPPP / PPP-K

$$\underbrace{\lambda_i^j(t_i)_{\text{Obs}} + D_i^j(t_i)_{\text{Obs}}}_{\text{Stored as "Phase-Observations" at } t_{i,R}} = \tilde{\rho}_i^j(t_i) + c \cdot \Delta \bar{t}_{R,i} - c \cdot \Delta \bar{t}_{S,j} + \Delta \rho_{i,\text{ION}}^j + \Delta \rho_{i,\text{TROP}}^j + \lambda \cdot (\Delta \phi_i + \Delta \phi_j - N_i^j)$$

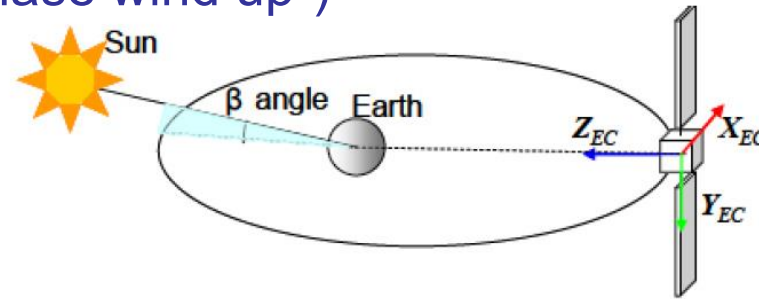
Stored as "Phase-Observations" at $t_{i,R}$

Uncalibrated phase delays (UPD) relevant to undifferenced integer fixing for PPP



Yaw-steering Mode

“Wind-up (“Phase wind up”)



Orbit-normal („Earth-centered“) Mode

Affects only carrier phase measurements (circularly polarized waves of GNSS signals).

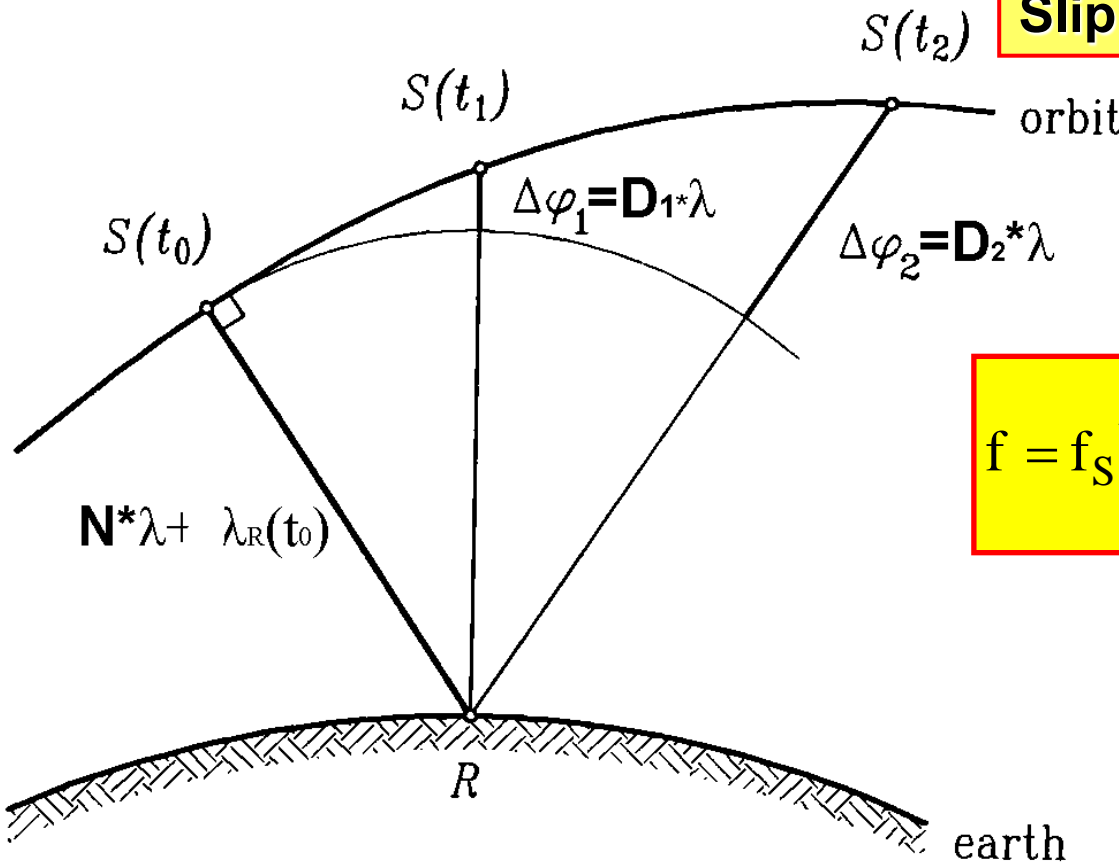
A correction $d\text{PHi}$ is only required for PPP absolute positioning.

$d\text{PHi}$ depends on relative orientation of satellite and receiver antennas, and the direction of the line of sight. While moving in the orbit the satellites perform a rotation to keep its solar panels pointing to the sun direction in order to obtain the maximum energy, while the satellite antenna keeps pointing to the earth's centre. This rotation causes the a phase variation $d\text{PHi}$.

http://www.navipedia.net/index.php/Carrier_Phase_Wind-up_Effect

Doppler-Frequency Measurement Types

$$D_i = \text{int}(\Delta\phi'_i [\text{cyc}]) = \text{int}\left(\int_{t_0}^{t_i} (f_R - f_S)^t dt\right)$$



1.) Add. Measurement for Positioning by Phase-Measurements

Cycles D_i at Phase-Measurements. Error: „Cycle-Slip(s)“

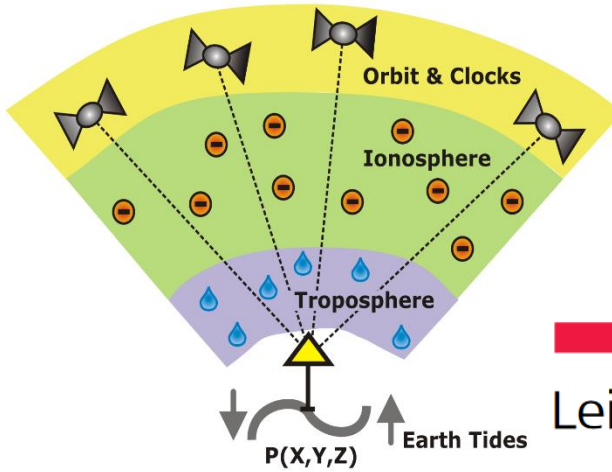
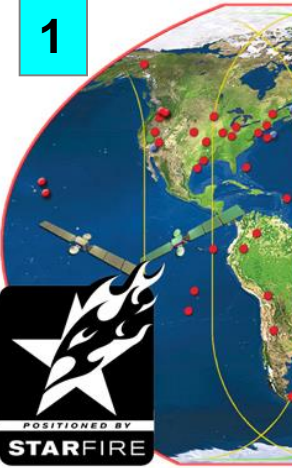
2.) General Measurement for Positioning and Navigation

$$f = f_S^t - f_R^t = \frac{f_S^t}{c} \cdot \frac{\mathbf{r}_S - \mathbf{r}_R}{|\mathbf{r}_S - \mathbf{r}_R|} \cdot (\dot{\mathbf{r}}_S - \dot{\mathbf{r}}_R)$$

$$\Delta f = f \cdot \left(\frac{1}{2} \cdot \left(\frac{v}{c} \right)^2 + \frac{\Delta U}{c^2} \right)$$

„Global G(lobal“)NSS Precise Positioning Services - Worldwide

1



- **SSR-based: Abs. Prec. OP**
- **Starfire™ GPS-Corrections**
- **Starfire Receiver (left)**
- **Global Accuracy: „dm“**



Leica **SteerDirect**
steering solutions

**Abs. GNSS = „Non-DGNSS“
No Reference-Stations
But: NAVCOM Roverclients!**



2 „RTX“

16 March 2011

Check out the Latest News!

[Read Press Release](#)

Continue to an OmniSTAR website:

- [North and South America](#)
- [Europe, North Africa, Middle East, India](#)
- [Asia Pacific](#)
- [South Africa](#)



Comparison of HP and other DGPS service characteristics

- Centimetre-level RTK
- Network RTK
- OmniSTAR-HP
- State Space Solution
- Wide Area DGPS

OmniSTAR-HP

- Uses range and phase data from a network of L1/L2 reference stations
- Ionospheric effects removed by using stations within 500-1000km
- Orbit corrections broadcast alongside data to remove this as an error source
- Multiple stations combined into L1/L2 VBS solution for redundancy of observations
- Smooth decimetre level positioning - with scope for further gains ?

**OSR (= Observation-)related: Networked, scalable („dm – cm“)
DGNS RTCM Correction (VRS-Concept)**

- **RTCM-Standard =>Open for any Rover- and Software-Type**

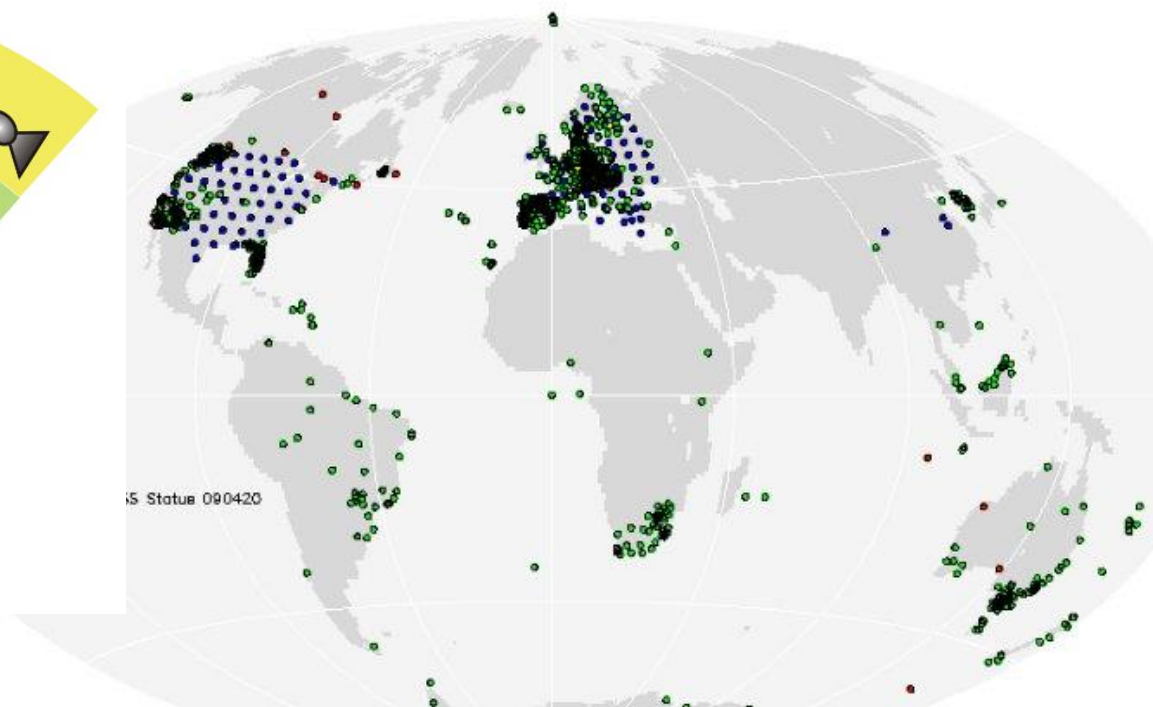
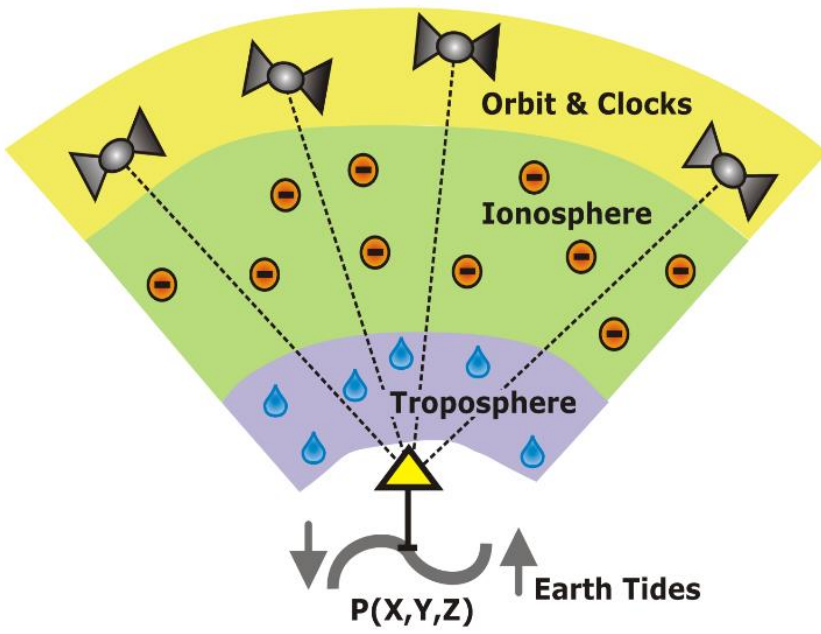
Commercial OPPP / PPP-K Services

NAME	Service	Accuracy performance	Supported constellations	Method	Provider
OmniStar	G2	<10cm	GPS+GLONASS	PPP	Trimble
RTX	CenterPoint	<4cm	GPS+GLONASS+BDS	PPP	Trimble
StarFix	G2+	3cm	GPS+GLONASS	PPP	Fugro
	G4	5-10cm	GPS+GLONASS+BDS+Galileo	PPP	
StarFire	SF2	5cm	GPS+GLONASS	PPP	John Deere
	SF3	3cm	GPS+GLONASS Future(BDS+Galileo)	PPP	
Veripos	C2	5cm	GPS+GLONASS	PPP	Hexagon AB
	Apex ²	5cm	GPS+GLONASS	PPP	
TerraStar	TerraStar C	2-3cm	GPS+GLONASS	PPP	Hexagon AB

August 2017: Joint Venture „Sapcorda Services“ (Bosch, Geo++, Mitsubishi, ublox)

International GNSS-Service (IGS)

„RTS“: <http://rts.igs.org/>



International GNSS Service

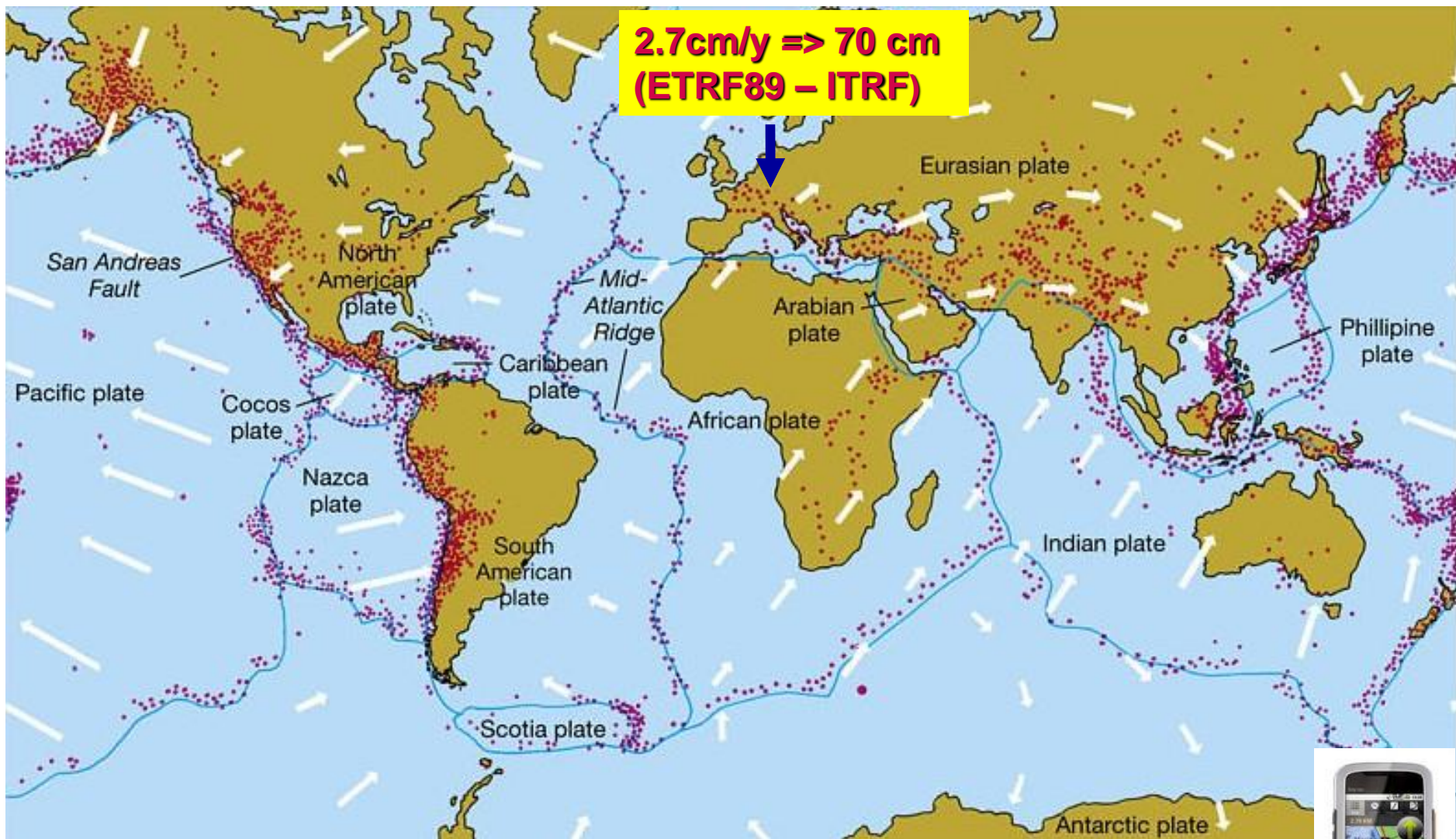
Formerly the International GPS Service

About	Products Mail	Network FAQ	Projects Publications	Events FTP	Organization Site map
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Real-time Service

[User Access](#) [Products](#) [RTS Monitoring](#) [Contributors](#) [More Information](#) [Support](#)

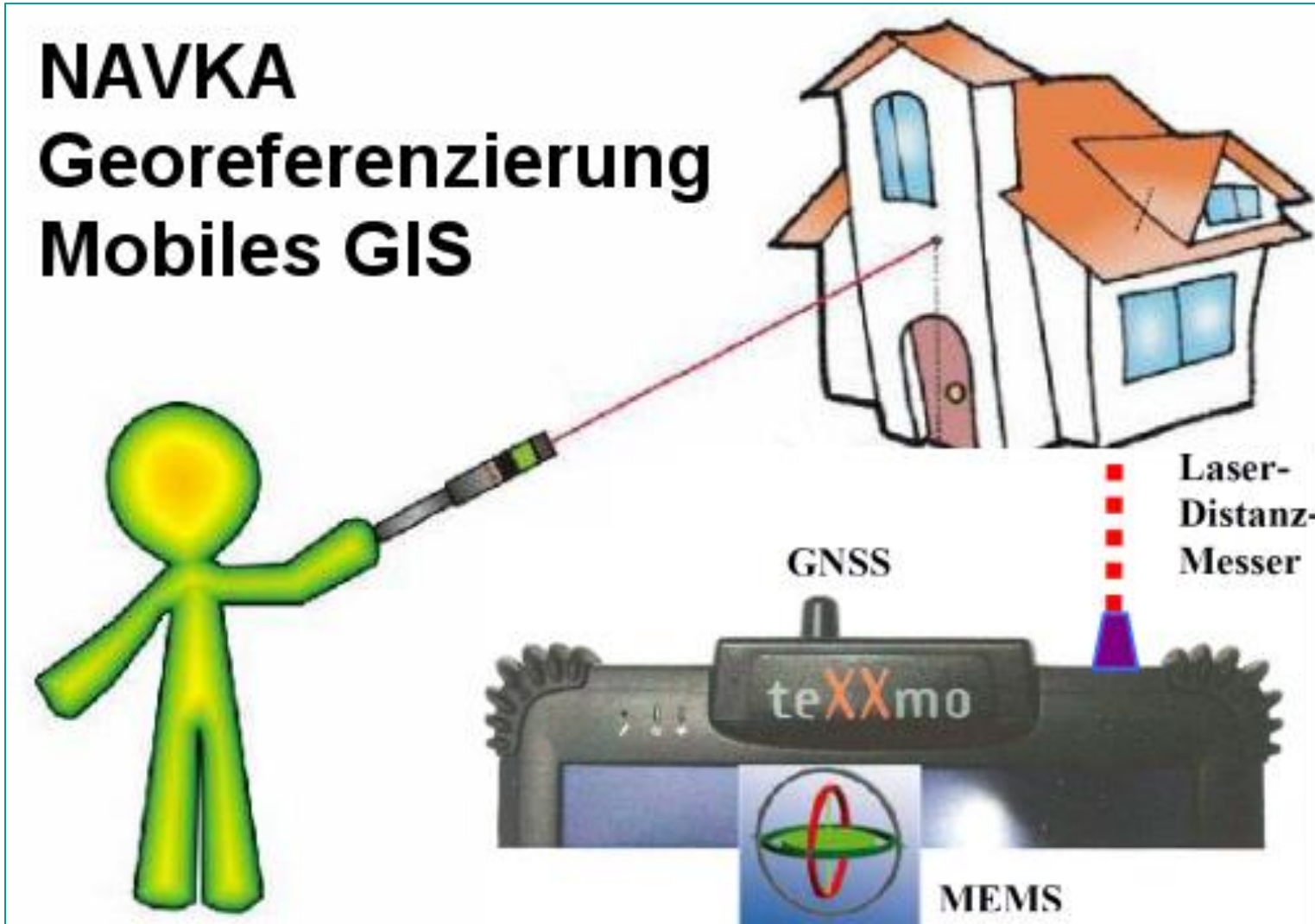
GNSS Positioning – Medium Term Geodynamical Modeling



$$\mathbf{x}(t_1)_{ITRF_{zz,t_1}} = (1 + \Delta m) \cdot \mathbf{R}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \cdot \mathbf{x}(t_1)_{ITRF_{yy,t_1}} + \mathbf{t}$$

$$\mathbf{x}(t_2)_{ITRF_{zz,t_2}} = \mathbf{x}(t_1)_{ITRF_{zz,t_1}} + \left(\left(\dot{\mathbf{R}} + \Delta \dot{m} \right) \cdot \mathbf{x}(t_1)_{ITRF_{zz,t_1}} + \dot{\mathbf{t}} \right) + \left(\dot{\mathbf{R}}_{P(j)} \cdot \mathbf{x}(t_1)_{ITRF_{zz,t_1}} \right) \cdot (t_2 - t_1)$$





PREGON-X RaD at HSKA:

<http://www.navka.de/index.php/de/weitere-projekte/fue-projekte-produkte-2b>

INSPIRE



NAVKA Indoor-Navigation – General Aspects

SMART CITIES, SMART UNIVERSITIES



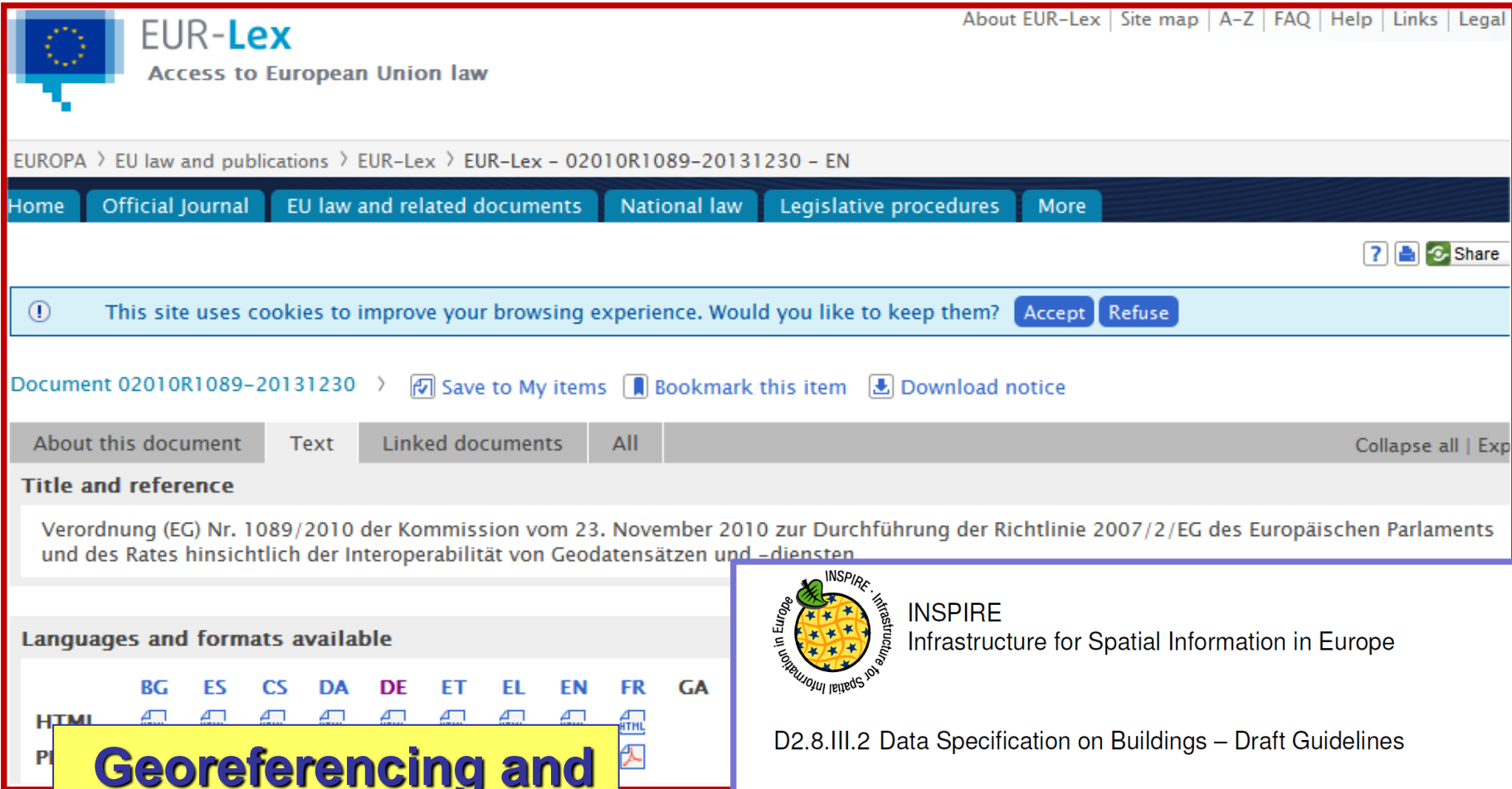
It's essential to develop technologies that allow us to improve the management of urban resources and efficiency in areas such as transport and traffic management, energy, health care, water and waste

The integration of these technologies is called **Smart cities**, and it allows to reduce costs and resource consumption and let governments take better and faster responses to urban challenges and issues.

- Cities cover only **2 %** of the Surface of the Earth,
- but they contain **50%** of the population,
- consume **75%** of the worlds energy and
- produce **80%** of the waste



NAVKA Indoor-Navigation – European Infrastructure INSPIRE



The screenshot shows the EUR-Lex website interface. At the top, there is a navigation bar with links for 'About EUR-Lex', 'Site map', 'A-Z', 'FAQ', 'Help', 'Links', and 'Legal'. The main header includes the EUR-Lex logo and the text 'Access to European Union law'. Below this, a breadcrumb trail indicates the document's location: 'EUROPA > EU law and publications > EUR-Lex > EUR-Lex - 02010R1089-20131230 - EN'. A secondary navigation bar contains buttons for 'Home', 'Official Journal', 'EU law and related documents', 'National law', 'Legislative procedures', and 'More'. A cookie consent banner is visible, asking if the user wants to accept cookies. Below the banner, there are options to 'Save to My items', 'Bookmark this item', and 'Download notice'. The document title is 'Verordnung (EG) Nr. 1089/2010 der Kommission vom 23. November 2010 zur Durchführung der Richtlinie 2007/2/EG des Europäischen Parlaments und des Rates hinsichtlich der Interoperabilität von Geodatensätzen und -diensten'. A section for 'Languages and formats available' lists various languages (BG, ES, CS, DA, DE, ET, EL, EN, FR, GA) and formats (HTML, PDF).

Georeferencing and Interoperability ITRF/ETRF89
<http://inspire.ec.europa.eu/index.cfm/pageid/3>



INSPIRE
Infrastructure for Spatial Information in Europe

D2.8.III.2 Data Specification on Buildings – Draft Guidelines

Title	D2.8.III.2 INSPIRE Data Specification on <i>Buildings</i> – Draft Guidelines
Creator	INSPIRE Thematic Working Group <i>Buildings</i>
Date	2012-04-20
Subject	INSPIRE Data Specification for the spatial data theme <i>Buildings</i>
Publisher	INSPIRE Thematic Working Group <i>Buildings</i>
Type	Text
Description	This document describes the INSPIRE Data Specification for the spatial data theme <i>Buildings</i>



Bundesanzeiger

Herausgegeben vom
Bundesministerium der Justiz

www.bundesanzeiger.de

Bekanntmachung

Veröffentlicht am Donnerstag, 14. November 2013
BAnz AT 14.11.2013 B1

Seite 1 von 17

Bundesministerium des Innern

Bekanntmachung
der Neufassung der Technischen Richtlinie zum Gesetz
über die geodätischen Referenzsysteme, -netze
und geotopographischen Referenzdaten des Bundes
(Technische Richtlinie Bundesgeoreferenzdatengesetz – TR BGeoRG)

Vom 28. Oktober 2013

<http://inspire-geoportal.ec.europa.eu/>

**Seamless Out-/Indoor Georeferencing and
Interoperability based on ITRF/ETRF89**

Paradigma-Changes GALILEO - Aspects

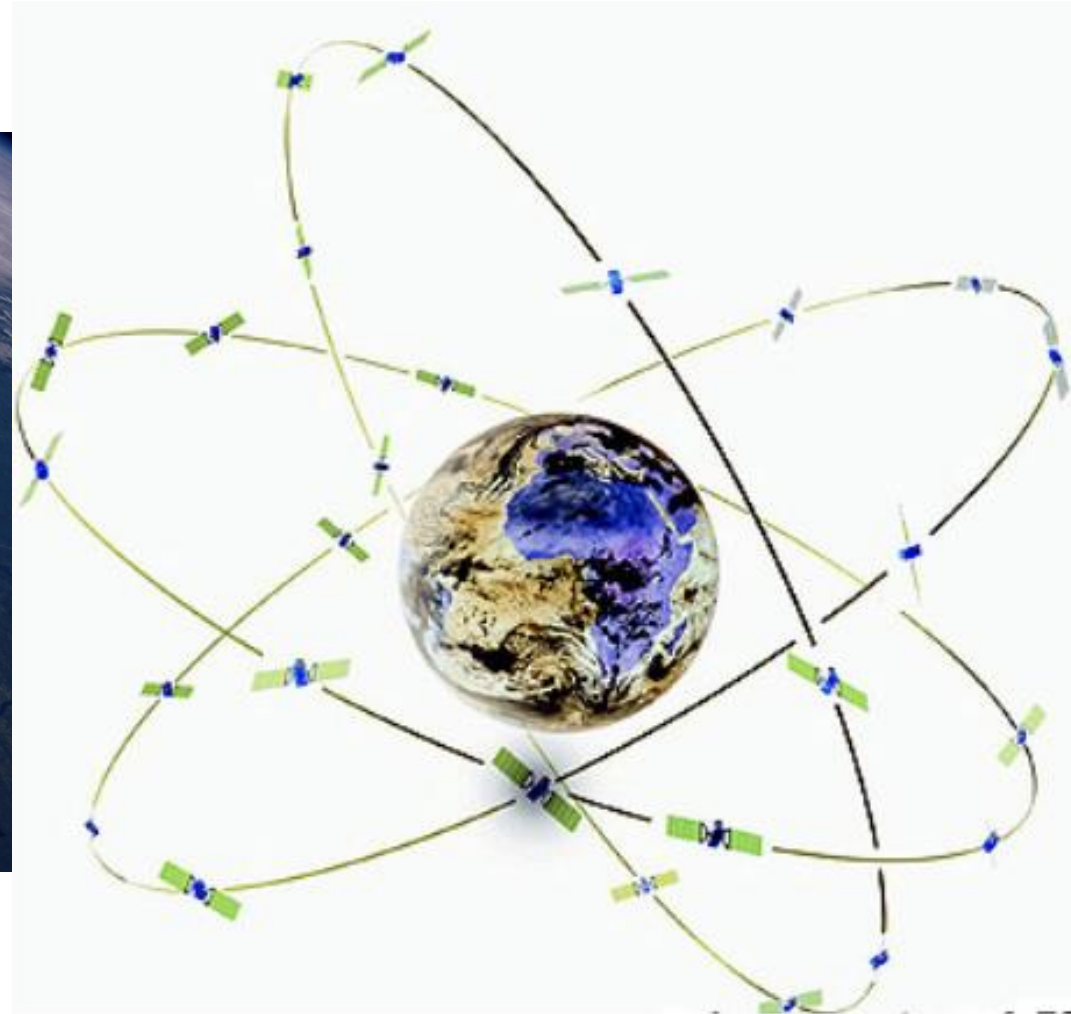


GALILEO

In Orbit Placement of GALILEO Satellites with Ariane 8 Satellites



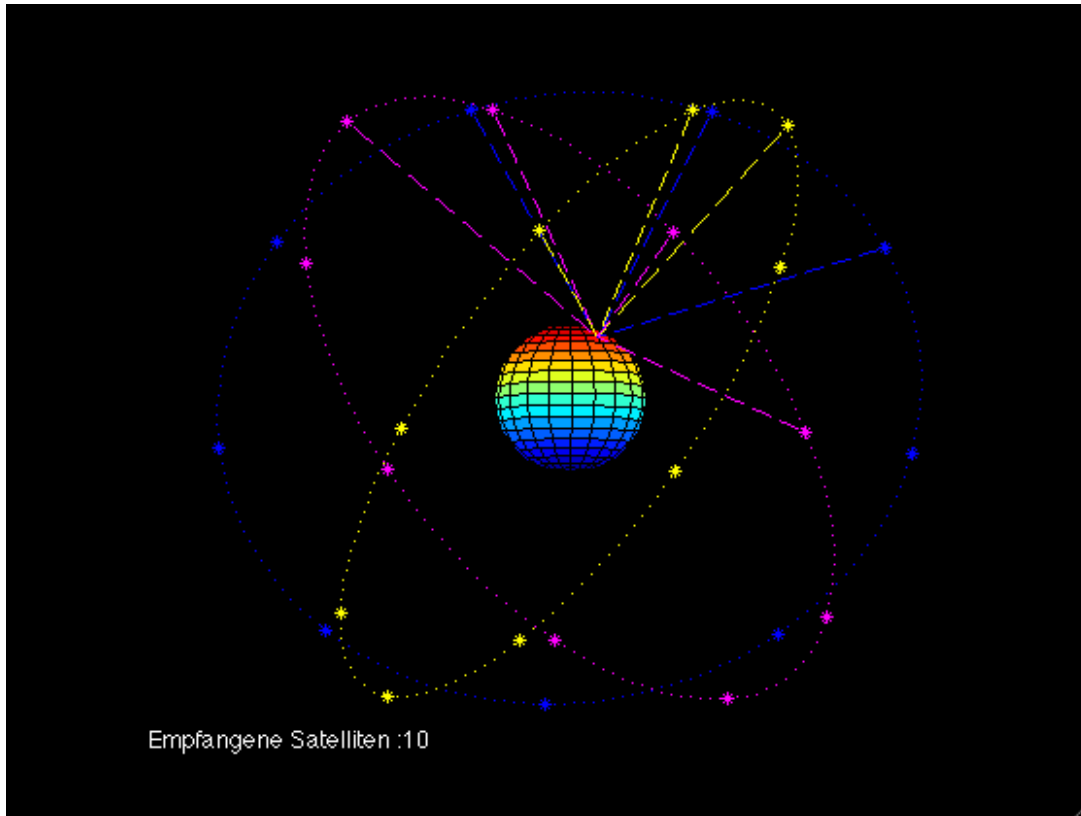
Galileo-Satellites



GALILEO and GPS Satellites

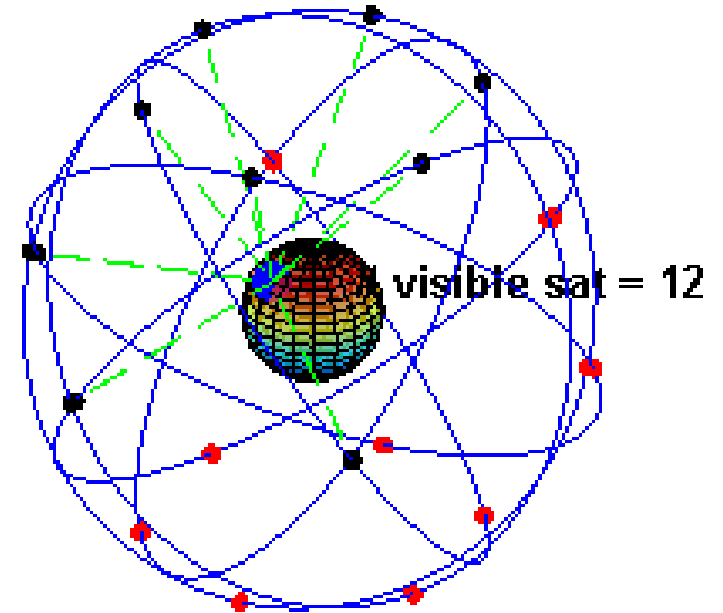
GALILEO Satelliten

https://en.wikipedia.org/wiki/List_of_Galileo_satellites

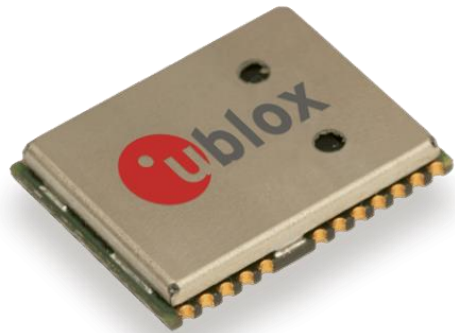


GPS Satelliten

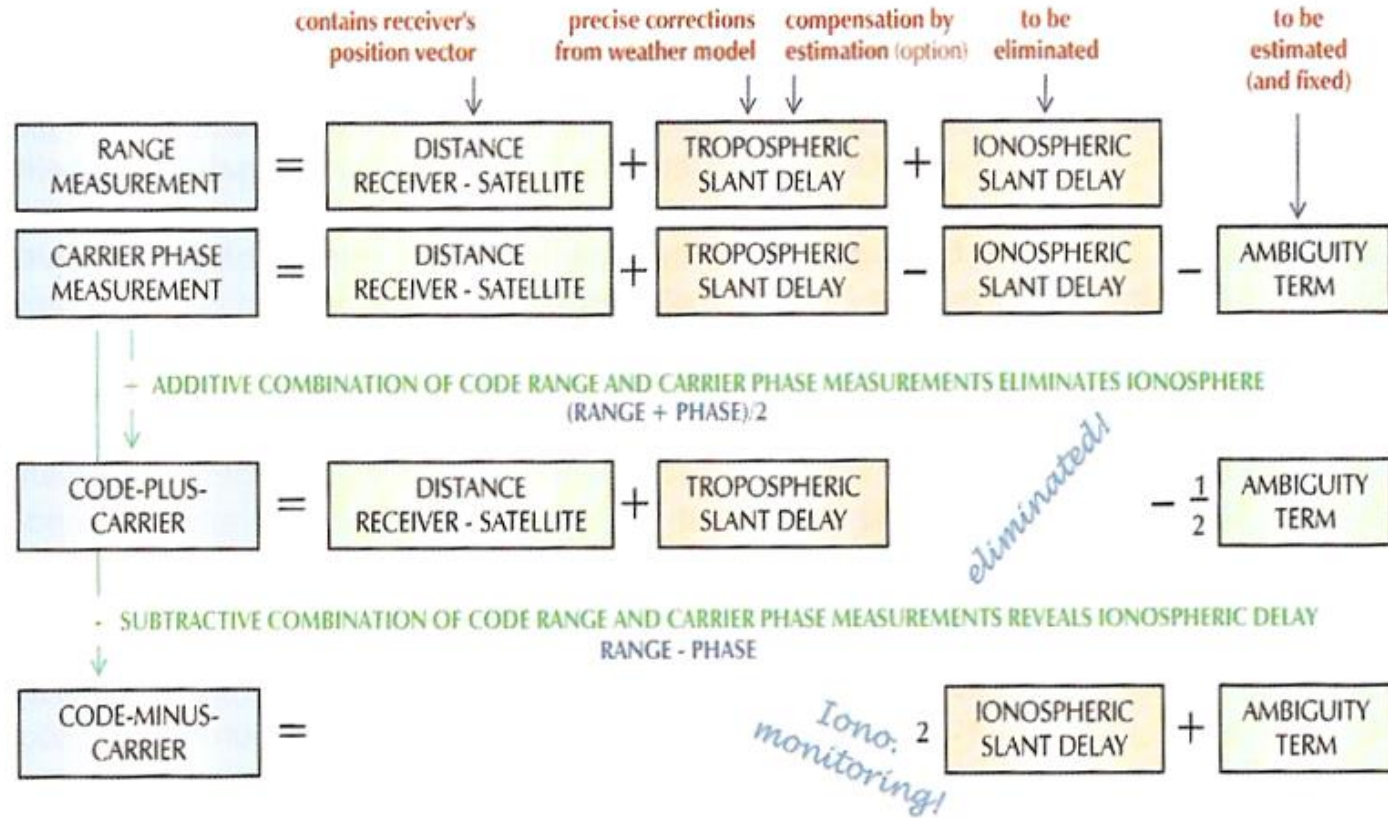
https://de.wikipedia.org/wiki/Global_Positioning_System



GALILEO - E1 Code/Phase Linear-Combination



Ublox NEO 8MN
29.- EUR



Vereinfachte Beobachtungsgleichung „Code-plus-Carrier“:

$$\frac{PR + \phi}{2} = \rho - \frac{\lambda}{2} \cdot N + \left(\frac{c_2}{4 \cdot f^3} + \frac{c_3}{3 \cdot f^4} \right) + SPD$$

kombinierte Beobachtung

Entfernung

Mehrdeutigkeit

Ionosphärenfehler
höherer Ordnung

troposphärische Laufzeit-
verzögerung (Slant Path
Delay)

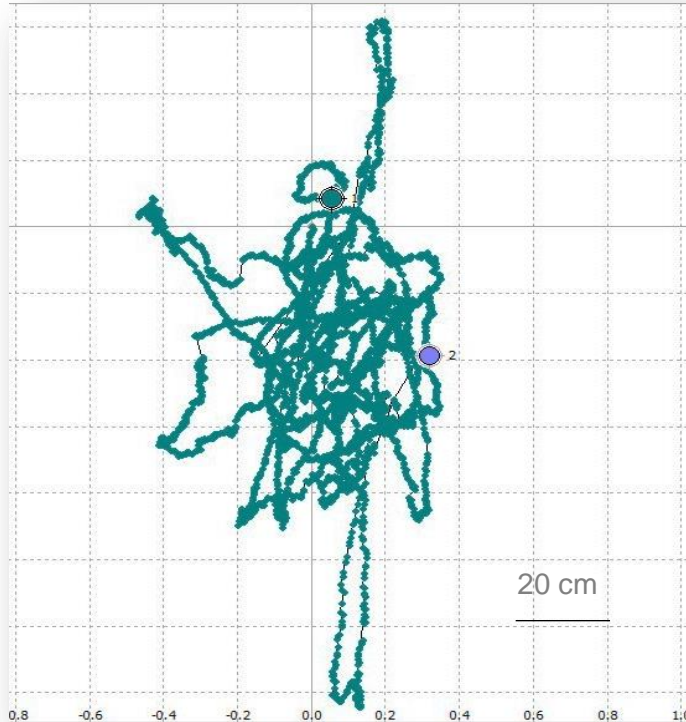
OPPP – L1-Pseudorange and –Phasemeasurements

RTS/IGS - Realtime Data Stream of IGS SSR-Products

▶ Rover Motion: Over Point 2

▶ Rover Motion: Around Point 2

L1, PPP-K



Key:

- ⊙ : Computed Position
- : Station's True Position found in "ITRF 2008" Frame

3D IGS/RTS OPPP Kinematic Positioning Error at 1 Hz in ITRF2008.09.2015
Positioning Error, Point 2: 0.25m



Broadcom launches dual-frequency GNSS receiver for mass market

September 21, 2017 - By Tracy Cozzens

**GNSS receiver
„BCM47755“**

1 cm

**OPP-K global
Positioning**

- GNSS-Receiver
- Smartphones

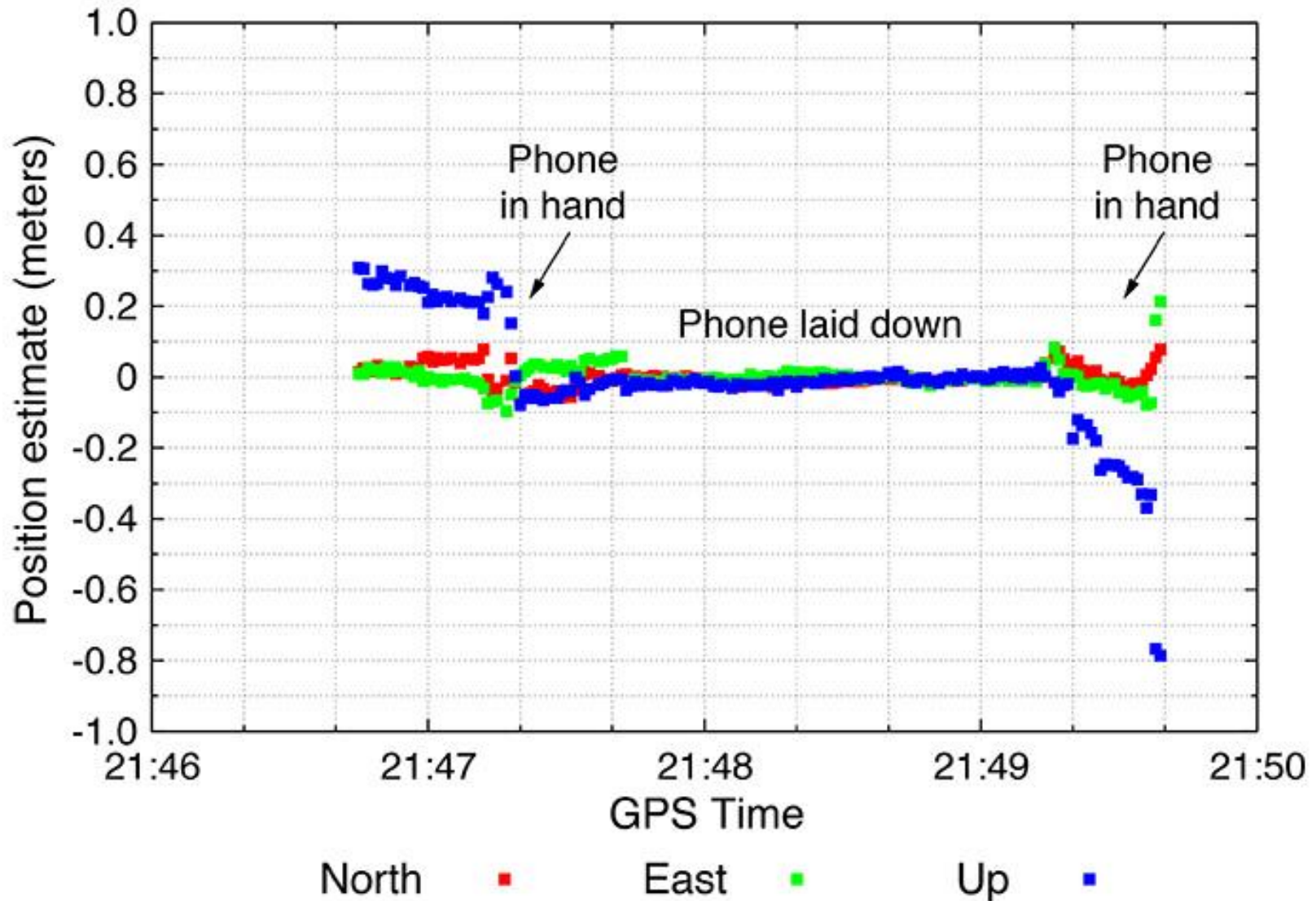
Open Source – DGNSS, PPP-K

RTKLIB - Open Source Software (DGNSS/RTK, GNSS-PPP, Postprocessing)

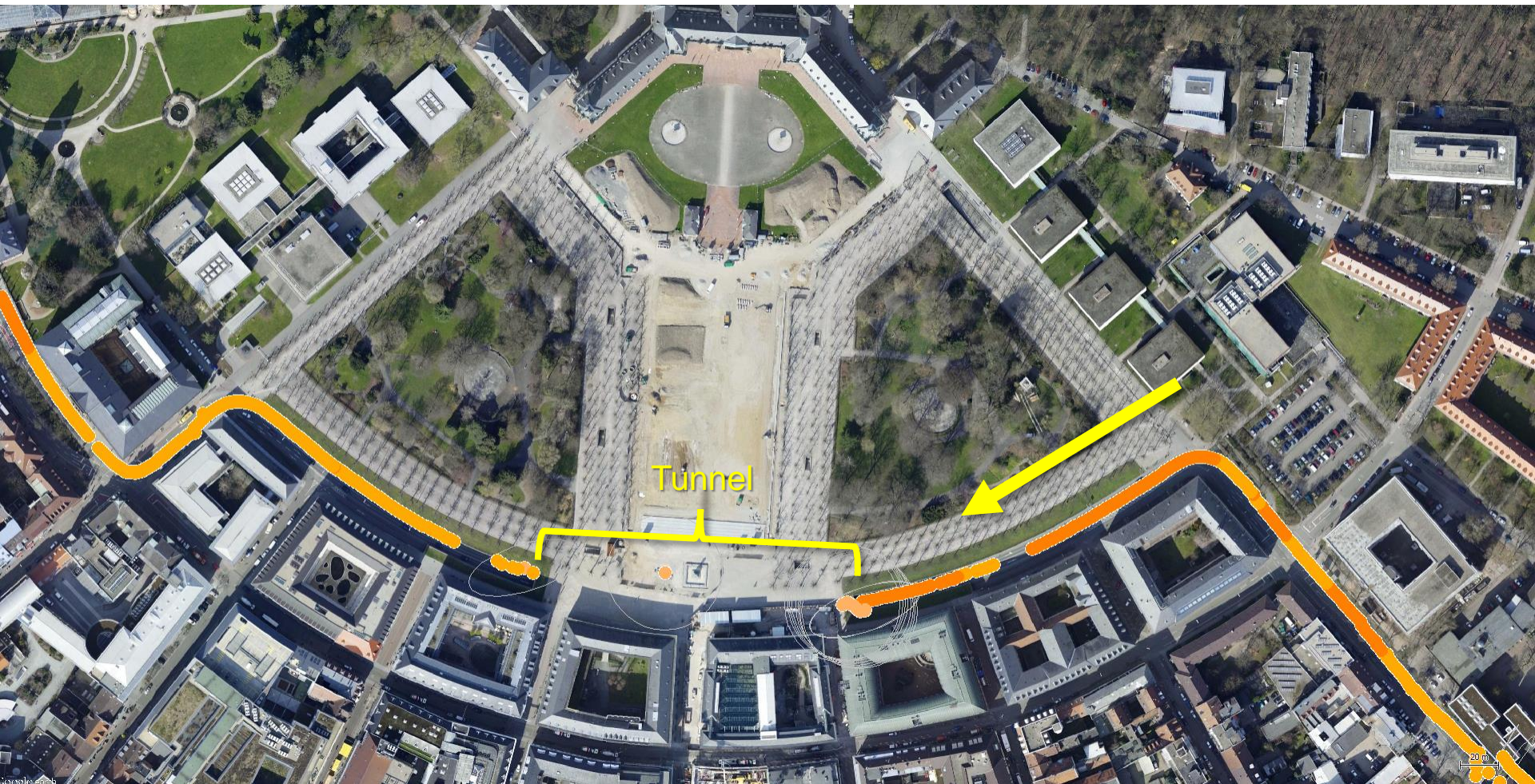
The screenshot displays the RTKLIB software interface, showing several windows and their contents:

- STRSVR ver.2.2.0:** Shows stream configuration with columns for Stream, Type, Out, and Bytes. It lists (1) Input: NTRIP Client and (2) Output: TCP Server.
- RTKNAVI ver.2.2.0:** Displays the solution type as SBAS and provides coordinates: N: 35° 52' 22.7486", E: 138° 23' 22.7875", H: 961.416 m. It also shows a bar chart representing signal strength or quality over time.
- RTKCONV ver.2.2.0:** Shows conversion settings for time (SPST) and receiver log files.
- Ntrip Source Table Browser:** Displays a table of NTRIP source stations. The table has columns: Hourpoint, ID, Format, Format-Decade, Cnt, and Nav-System/network. The table lists various stations like ACSI0, ACSI1, A.D.H, etc.
- RTKPOST ver.2.2.0:** Shows post-processing settings, including observation data and navigation messages.
- RTKNAVI (Map):** Shows a map with a green trajectory line, indicating the path of the receiver. The map includes a scale bar (0 to 100 meters) and a north arrow.
- About windows:** Several small windows provide information about the software modules: STRSVR, RTKNAVI, RTKCONV, Ntrip Source Table Browser, RTKPOST, and RTKPLOT, all with copyright information for T. Takasu.

GNSS Rawdata on Smartphones (since Sept. 2016)



New Way (NAVKA): Algorithms for Multiplatform-, Multisensor-, Leverarm-Design



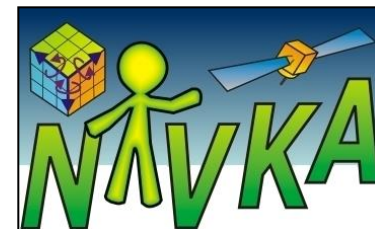
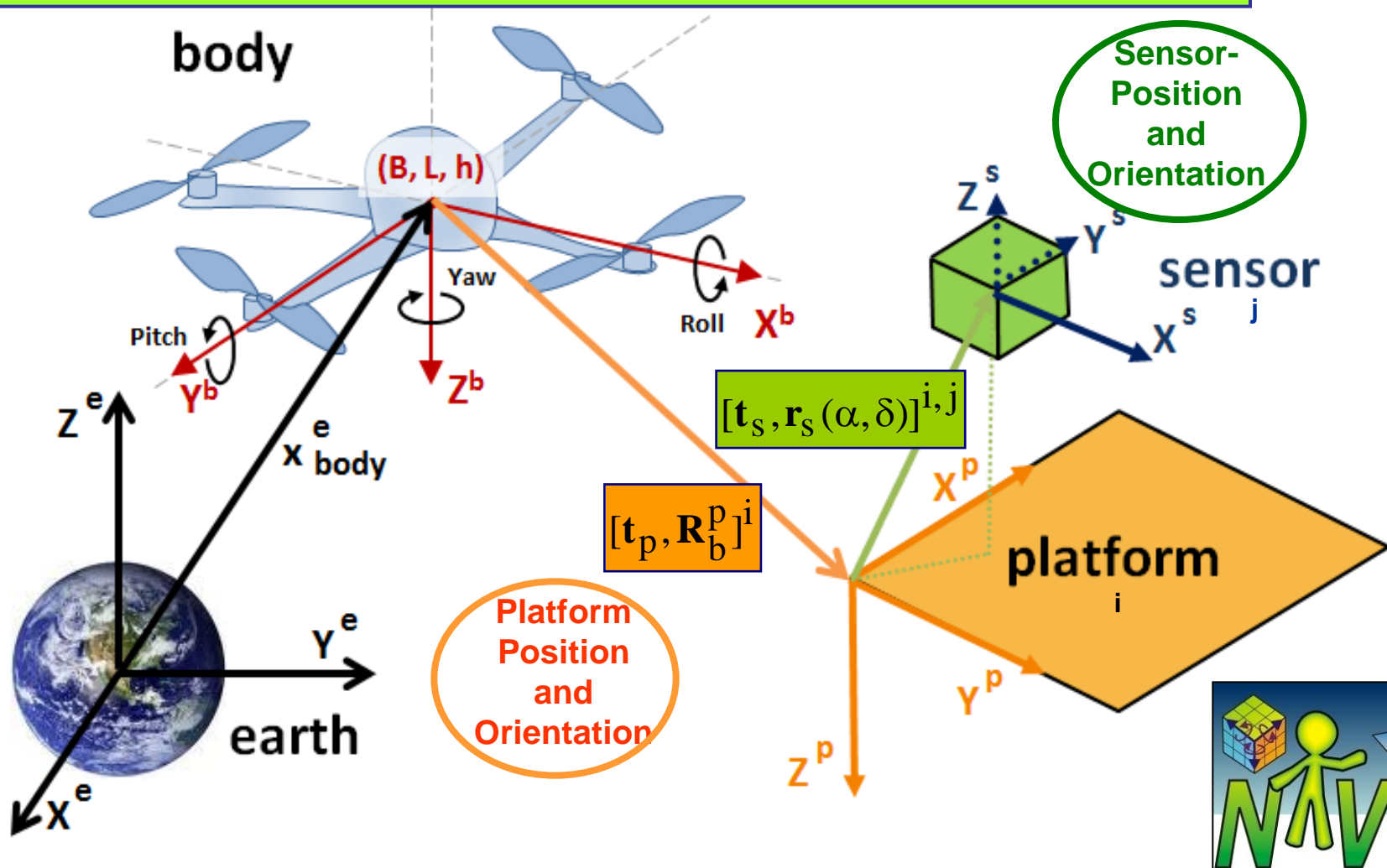
- GNSS Positioning Single L1 Receiver

Paradigma-Changes **MULTISENSOR-INTEGRATION**



$$y = \left[x^e \ y^e \ z^e \mid \dot{x}^e \ \dot{y}^e \ \dot{z}^e \mid r^e \ p^e \ y^e \parallel \ddot{x}^e \ \ddot{y}^e \ \ddot{z}^e \mid \omega_{eb,x}^b \ \omega_{eb,y}^b \ \omega_{eb,z}^b \mid \mathbf{s} \right]^T$$

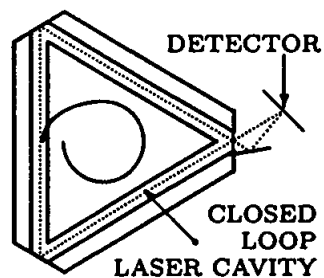
3.) „Multisensor-Multiplatform Leverarm“ Concept



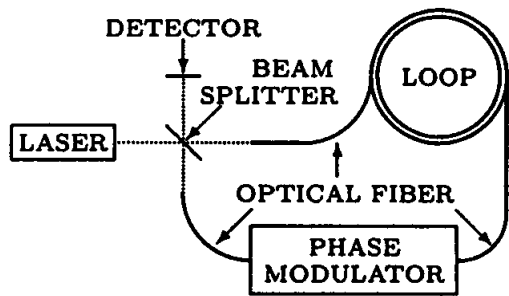
Autonomous MEMS-Sensors + New Algorithms „Deep-Coupling“

2. Autonomous

1.) MEMS-Gyro

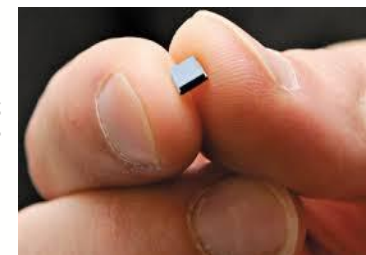
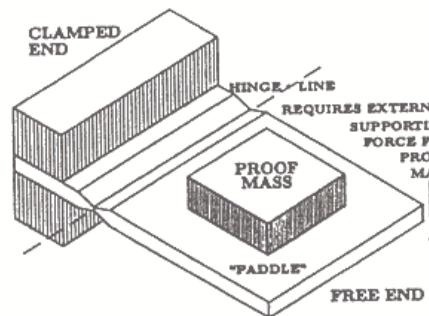


(a) Ring Laser Gyro (RLG)



(b) Fiber Optic Gyro (FOG)

2.) MEMS-Accelerometers



Observation I: ω_{ip}^p

$$\omega_{ep}^p = \omega_{ip}^p - \mathbf{R}_e^p(r, p, y) \cdot \omega_{ie}^e$$



Observation I: \mathbf{a}^p

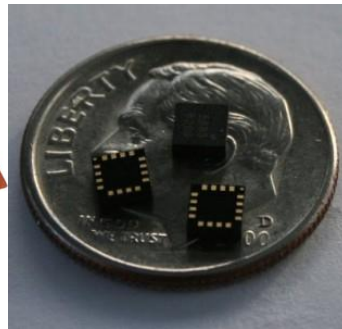
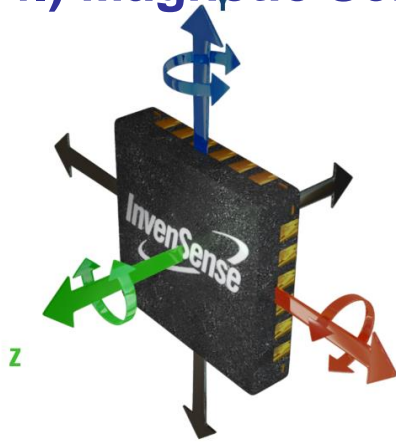
$$\ddot{\mathbf{x}}^i(t) = \mathbf{g}^i(\mathbf{x}) + \mathbf{R}_p^i(t) \cdot \mathbf{a}^p$$

References: Inertial Space (i) and Gravity Field

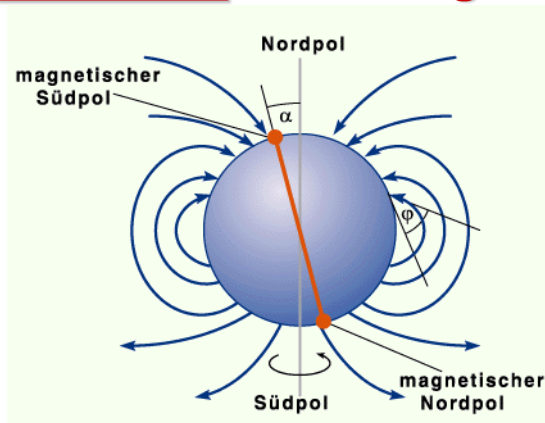


2. Autonomous

4.) Magnetic Sensors



Reference: Earthmagnetic Field M

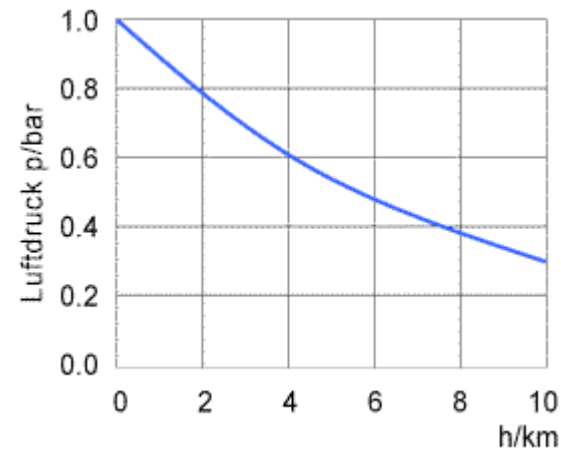


5.) Barometric Sensors (Height)

Auxiliary Sensor

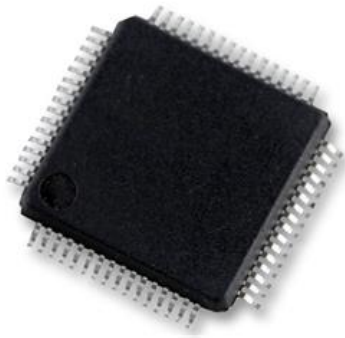


Reference: Earth Atmosphere



CPU and GNSS/MEMS Sensors e.g. in FlightControl FC4

microcontroller
Cortex M4 (ARMv7-M)
ST Microelectronics F4 family



9 DOF
accelerometer
gyroscope
magnetometer



**InvenSense
MPU-9150**

barometer



**Measurement
Specialities MS5611**

telemetry



LAIRD RM-024

GPS



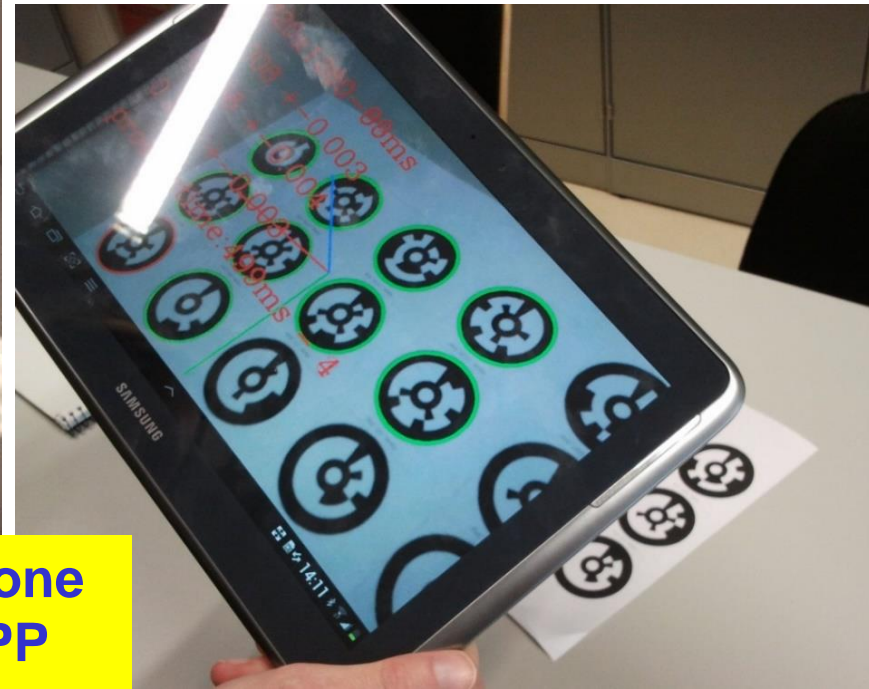
u-blox NEO-M8N

accelerometer
(thermal MEMS)



MEMSIC MXR-9500

NAVKA Camera based Indoor-Navigation-Concepts



**SMART-Phone
Marker APP
(Optical
Markers, LED,
Infrared)**



$$u_i = cx + \left(\frac{r_{11} * (X_i - X_0) + r_{12} * (Y_i - Y_0) + r_{13} * (Z_i - Z_0)}{r_{31} * (X_i - X_0) + r_{32} * (Y_i - Y_0) + r_{33} * (Z_i - Z_0)} \right) * f$$

$$v_i = cy + \left(\frac{r_{21} * (X_i - X_0) + r_{22} * (Y_i - Y_0) + r_{23} * (Z_i - Z_0)}{r_{31} * (X_i - X_0) + r_{32} * (Y_i - Y_0) + r_{33} * (Z_i - Z_0)} \right) * f$$



Cameras of Digital Smartphones / Tablet PC Infrastructure-based (“Virtual Markers”)

$$u_i = cx + \left(\frac{r_{11} * (X_i - X_0) + r_{12} * (Y_i - Y_0) + r_{13} * (Z_i - Z_0)}{r_{31} * (X_i - X_0) + r_{32} * (Y_i - Y_0) + r_{33} * (Z_i - Z_0)} \right) * f$$

$$v_i = cy + \left(\frac{r_{21} * (X_i - X_0) + r_{22} * (Y_i - Y_0) + r_{23} * (Z_i - Z_0)}{r_{31} * (X_i - X_0) + r_{32} * (Y_i - Y_0) + r_{33} * (Z_i - Z_0)} \right) * f$$

“Virtual Landmarks (VLM)”

**NAVKA Smartphone- and Tablet
Indoor Navigation with Virtual
Markers identified by Camera**

&

**Deep Coupling with all other MEMS-
Smartphone Sensors**

&

**Infrastructure Position-
Information (e.g. RFID and other)**

&

Indoor Mapmatching

& ...



**SMART-Phone Marker
APP (Optical Markers,
LED, Infrared)**

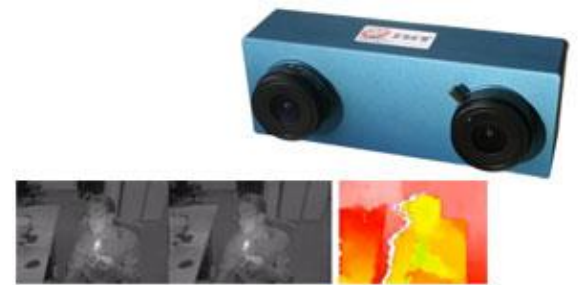
1.)

Velocity - Observation from the differentiation of subsequent VO-based position-differences

$$\mathbf{v}^{VO}(t)$$

General leverarm-situation for Stereo-Camera

$$\mathbf{t}_{VO}^b \quad \mathbf{R}(r_x, r_y, r_z)_b^{VO}$$



Final Observation Equation in Tight Coupling

$$\mathbf{v}^{VO}(t) = \mathbf{R}_b^{VO} \cdot \mathbf{R}(r, p, y)_e^b \cdot (\dot{\mathbf{x}}(t)^e + \mathbf{R}(r, p, y)_b^e \cdot (\boldsymbol{\omega}_{eb}^b \times \mathbf{t}_{VO}^b))$$

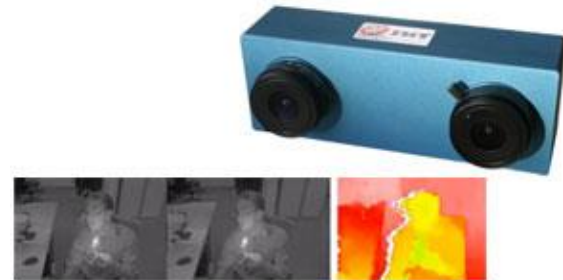
2.)

Orientation-rate observation from the subsequent VO-based orientation change

$$\Omega_{e,VO}^{VO}(t)$$

General leverarm-situation for Stereo-Camera

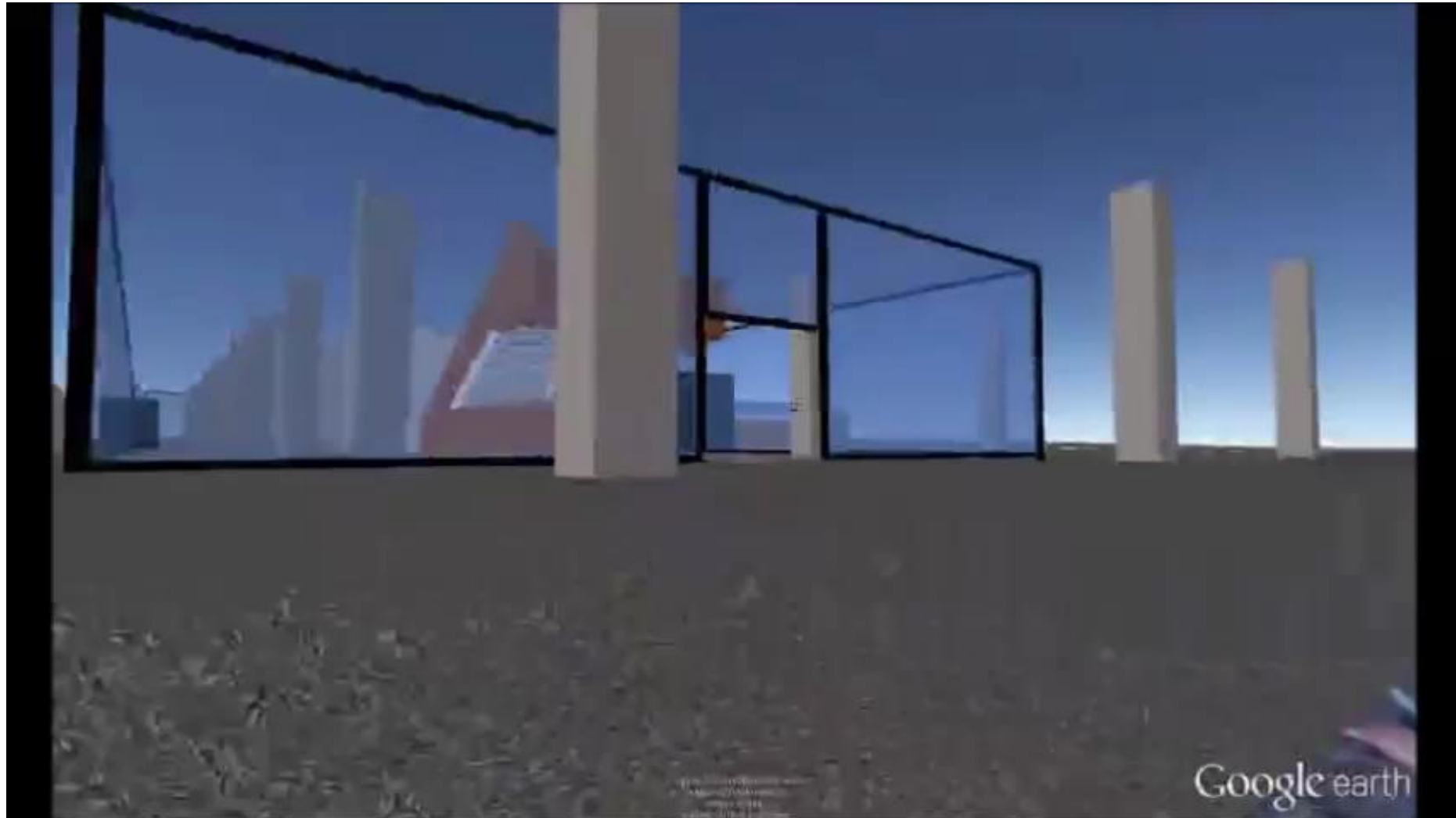
$$\mathbf{t}_{VO}^b \quad \mathbf{R}(r_x, r_y, r_z)_b^{VO}$$



Final Observation Equation in Tight Coupling

$$\begin{aligned} \Omega_{\text{"Object-Frame"},VO}^{VO}(t) &= \mathbf{R}_b^{VO} \cdot (\Omega_{\text{Object-Frame},VO}^b(t)) \cdot (\mathbf{R}_b^{VO})^T \\ &= \mathbf{R}_b^{VO} \cdot (\Omega_{\text{Object-Frame},e}^b(t) + \Omega_{e,b}^b(t) + \Omega_{b,VO}^b(t)) \cdot (\mathbf{R}_b^{VO})^T \end{aligned}$$

NAVKA Seamless Out-/Indoor-Navigation-Concepts



<https://www.youtube.com/watch?v=FvesMeAF3HY>

Autonomous Out-/Indoor Navigation – „NAVKArine G1MC“



Hochschule Karlsruhe
Technik und Wirtschaft
UNIVERSITY OF APPLIED SCIENCES



Gefördert durch:



aufgrund eines Beschlusses
des Deutschen Bundestages

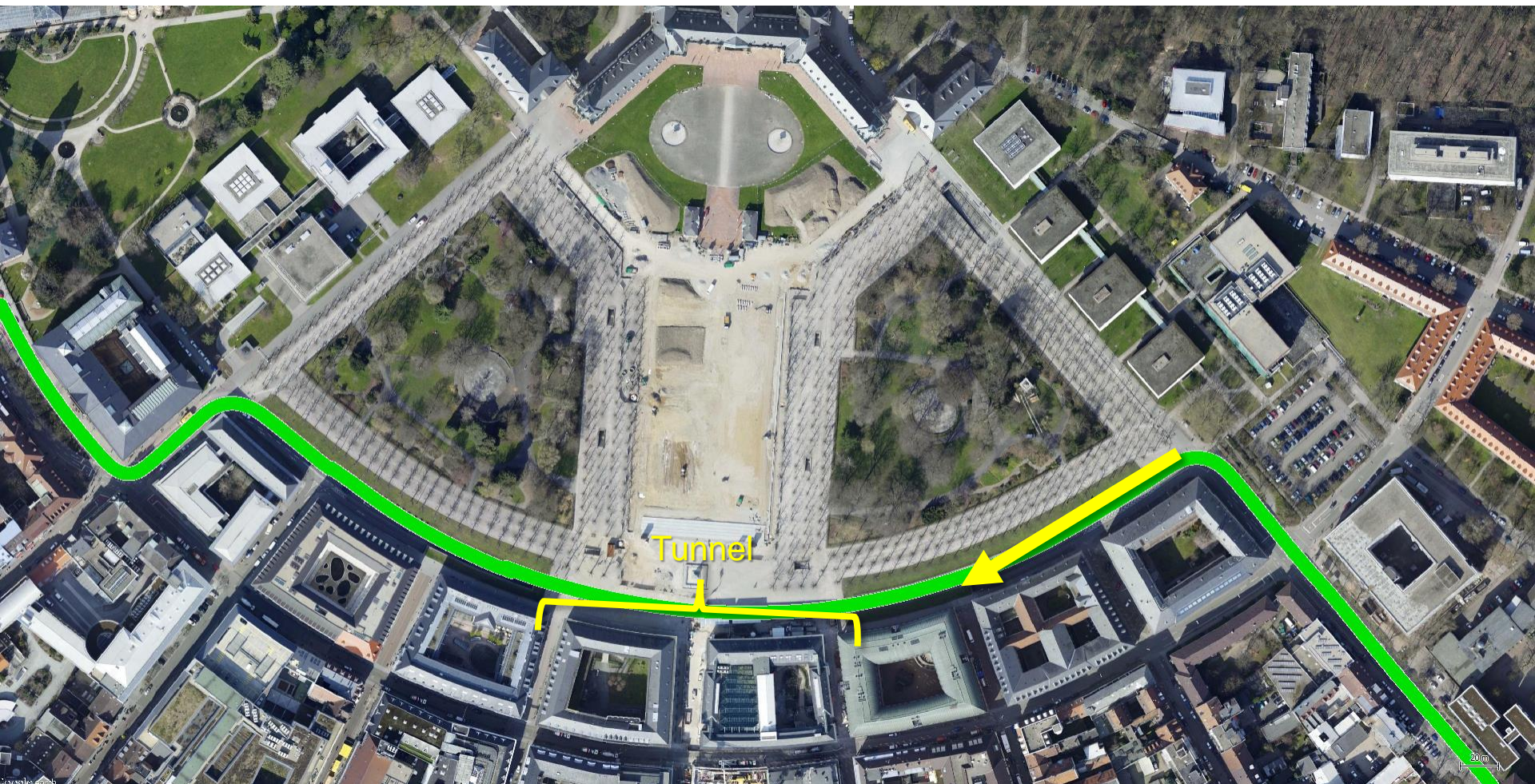
NAVKArine G1MC

Multisensor Navigationsplattform



210

New Way (NAVKA): Algorithms for Multiplatform-, Multisensor-, Leverarm-Design



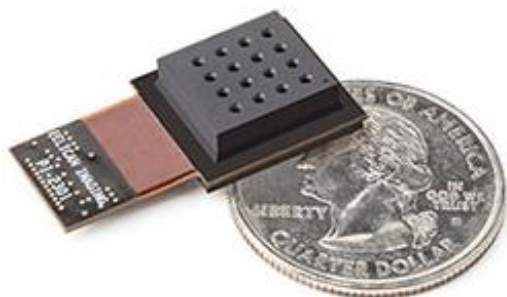
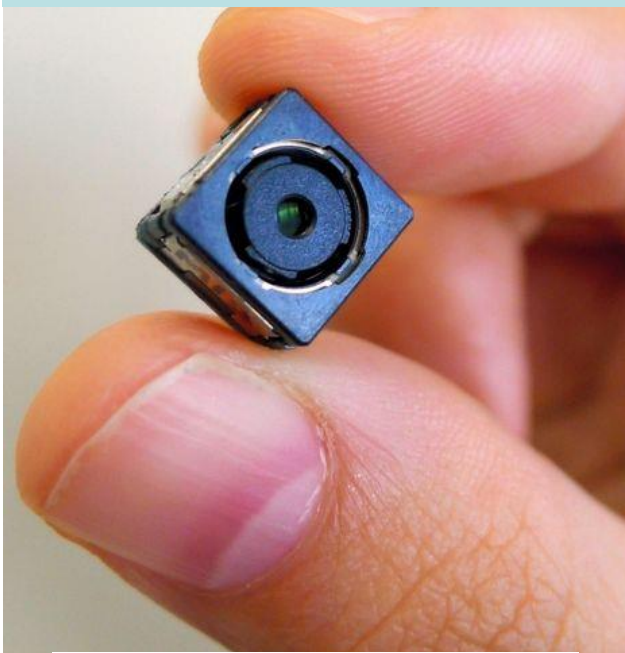
- NAVKA-Sensorfusion (GNSS/MEMS/MOMS) - Full 3D Navigation State-Information (Position, Velocity, Acceleration, Attitude, Rotationrate-Rate) Information

<https://www.youtube.com/watch?v=ymuhJ6pt52o>

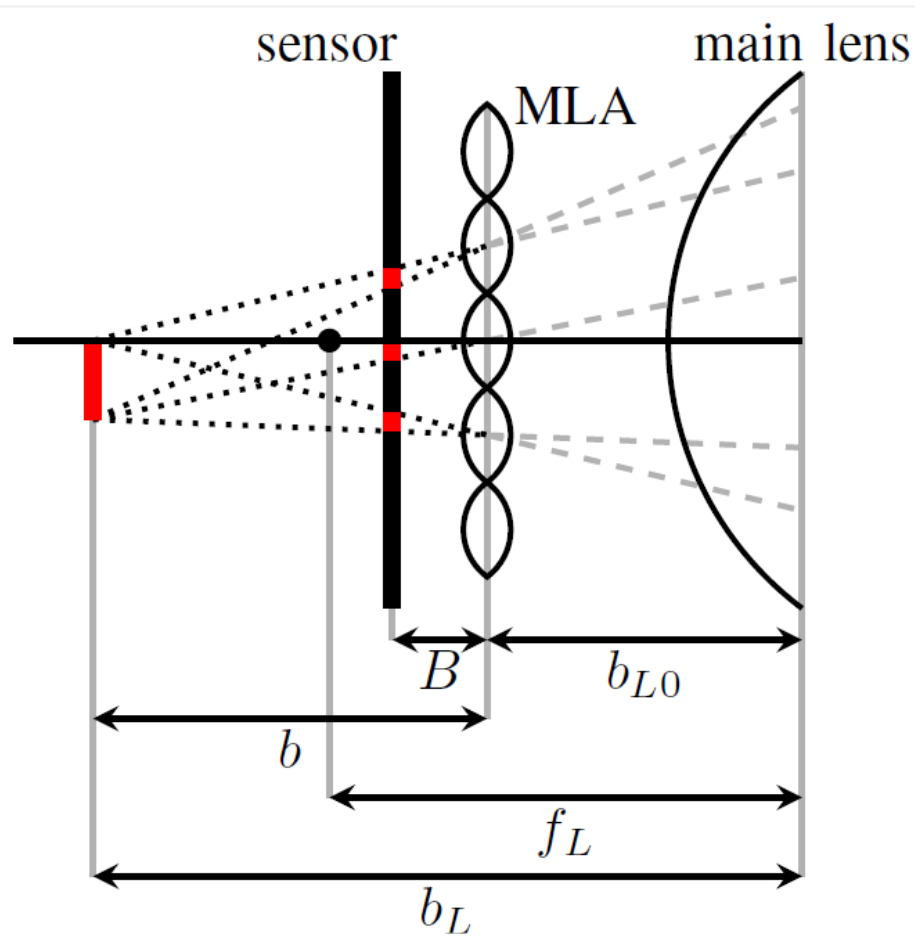
NAVKA Seamless Out-/Indoor-Navigation-Concepts

MOEMS – Plenoptic Cameras

1 cm x 1 cm x 1cm © Toshiba



Pelican Imaging 3D-Viewer



Principle: Main Lense and MLA

GNSS/MEMS/MOEMS based State Estimation



State Estimation

$$\text{bel}(\mathbf{y}_t) = p(\mathbf{y}_t / \mathbf{y}_0, \mathbf{l}_{0:t}, \mathbf{u}_{0:t})$$

1.) Stepwise Prediction and Control, Error \mathbf{s} , arbitrary density p_s

$$\mathbf{y}_t + \mathbf{s} = \mathbf{T}(\mathbf{y}_{t-1}, \mathbf{u}_t)$$

2.) Stepwise Measurements \mathbf{l}_t concerning the statevektor \mathbf{y}_t with measurement error \mathbf{e} , arbitrary density p_e

$$\mathbf{l}_t + \mathbf{e} = \mathbf{l}(\mathbf{y}_t)$$

3.) Arbitrary starting statevektor $\mathbf{y}_{t=0}$ and density \mathbf{p}_{y_0}

General Concept: Recursive Bayes-Estimation and 1. Order Markov

$$\underbrace{p(\mathbf{y}_t / \mathbf{y}_0, \mathbf{l}_{0:t}, \mathbf{u}_{0:t})}_{\text{A-posteriori Density}} = \eta \cdot \underbrace{p(\mathbf{l}_t | \mathbf{y}_t)}_{\text{Sensor Measurements Density}} \cdot \int_{-\infty}^{+\infty} \underbrace{p(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{u}_t)}_{\text{Prediction-Density } \mathbf{t}_t} \cdot \underbrace{p(\mathbf{y}_{t-1} | \mathbf{l}_{0:t-1}, \mathbf{u}_{0:t-1})}_{\text{A-priori Density}} \cdot d\mathbf{y}_{t-1}$$

Chapman-Kolmogorov-Equation for Prediction of State from t-1 to t

$$p(\mathbf{y}_t | \mathbf{y}_0, \mathbf{l}_{0:t-1}, \mathbf{l}_t, \mathbf{u}_{0:t-1}, \mathbf{u}_t, \mathbf{y}_{t-1}) = \int_{-\infty}^{+\infty} p(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{u}_t) \cdot p(\mathbf{y}_{t-1} | \mathbf{l}_{0:t-1}, \mathbf{u}_{0:t-1}) \cdot d\mathbf{y}_{t-1}$$

GNSS/MEMS/MOEMS based State Estimation

1. Component: State Transition of the Body (b) in regard



$$y = \left[x^e \ y^e \ z^e \mid \dot{x}^e \ \dot{y}^e \ \dot{z}^e \mid r^e \ p^e \ y^e \mid \ddot{x}^e \ \ddot{y}^e \ \ddot{z}^e \mid \omega_{eb,x}^b \ \omega_{eb,y}^b \ \omega_{eb,z}^b \mid \mathbf{s} \right]^T$$

1.) State Transition-Equations for the body (b) in the e-frame

Space Curve of the body (b) in the e-frame

$$\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \\ \ddot{\mathbf{x}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & [\Delta t] & \left[\frac{1}{2} \Delta t^2 \right] \\ \mathbf{0} & \mathbf{I} & [\Delta t] \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}(t - \Delta t) \\ \dot{\mathbf{x}}(t - \Delta t) \\ \ddot{\mathbf{x}}(t - \Delta t) \end{bmatrix}$$

Modification
of the State
Parameters
and
Equations

by

Rotation Rates of the Body (b) with respect to the e-frame

$$\mathbf{\Omega}_{eb}^b(t) = \mathbf{\Omega}_{eb}^b(t - \Delta t)$$

Considering
Special Conditions
Detected in
Multithreading
Computing

Orientation / Attitude

$$\mathbf{R}_e^b(t) = \mathbf{R}_e^b(t - \Delta t) \cdot \left[\mathbf{I} + \mathbf{\Omega}_{eb}^b \cdot \Delta t + \frac{1}{2!} \cdot (\mathbf{\Omega}_{eb}^b)^2 \cdot \Delta t^2 + \dots \right] \text{ with } \mathbf{\Omega}_{eb}^b = \mathbf{\Omega}_{ib}^b(\text{Sensor}) - \mathbf{\Omega}_{ie}^b$$

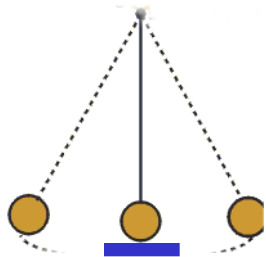
Further NAVKA-Key-Characteristics

$$\mathbf{x} = \left[x^e \ y^e \ z^e \mid v_x^e \ v_y^e \ v_z^e \mid r^e \ p^e \ y^e \parallel \ddot{x}^e \ \ddot{y}^e \ \ddot{z}^e \mid \omega_{eb,x}^b \ \omega_{eb,y}^b \ \omega_{eb,z}^b \right]^T$$

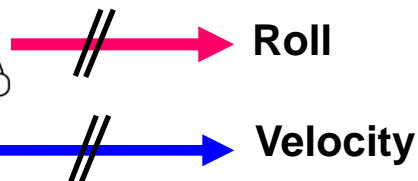
2.) General M-Estimation & Additional State Information

2.1) Parallel Processing Algorithms

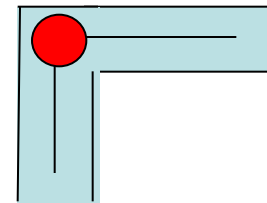
$$\mathbf{F}(\mathbf{x}) = \mathbf{0}$$



Zero-Updates („ZUPT“)



„Automotive Mode“



„Indoor-Map-Matching“

2.2) Condition In-/Equations

$$\mathbf{u} \leq \mathbf{F}(\mathbf{x}) \leq \mathbf{0}$$

SIMPLEX Algorithms
Providing also
Robustness via
L1-Norm based
M-Estimation

2.3) Sensorintegration



GNSS/MEMS/MOEMS based State Estimation

Sensors providing Space / Parameter Relation



Accelerometer- Sensorobservation j (Rawdata, on i -th Platform (p) – one-dimensional!)

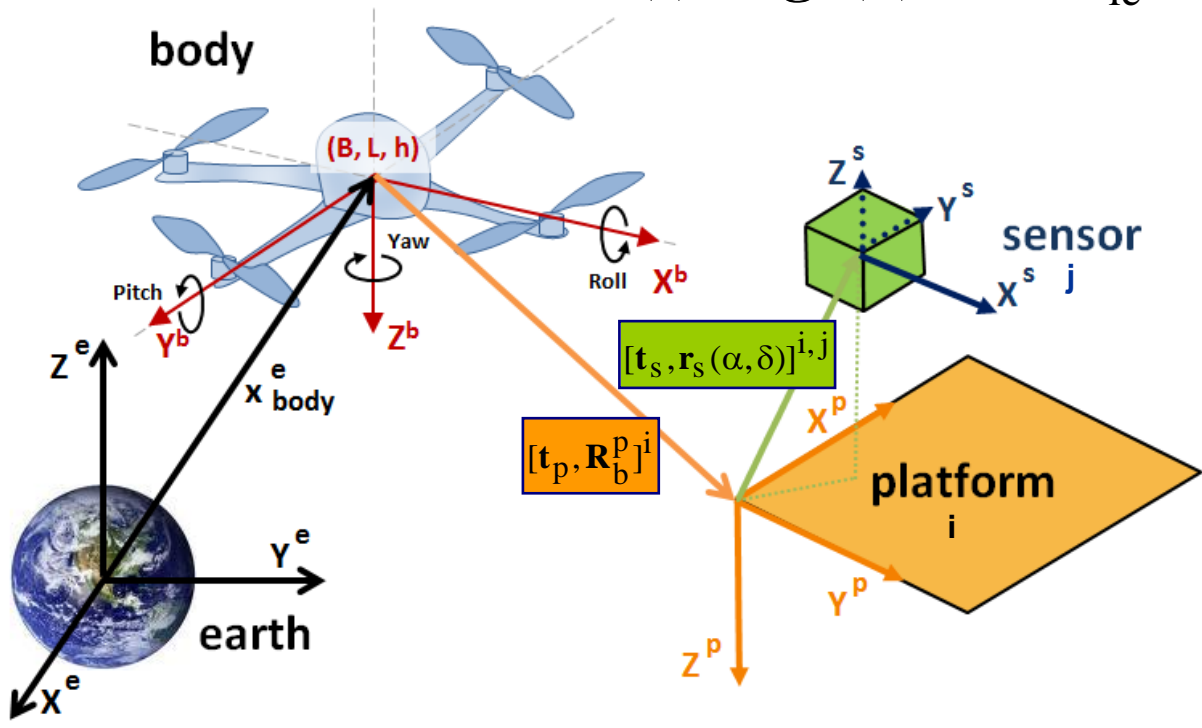
$$a_{S_{i,j}} = \mathbf{r}_{S_{i,j}}^{P_i} \cdot \mathbf{a}_{S_{i,j}}^{P_i}$$

$$\mathbf{a}_{S_{i,j}}^{P_i} = \mathbf{R}_b^{p,i} \cdot \mathbf{R}_e^b(r, p, y) \cdot \mathbf{a}_{S_{i,j}}^{e,i}$$

$$a_{S_{i,j}} = \mathbf{r}_{S_{ij}}^{P_i} \cdot \mathbf{R}_b^{p,i} \cdot \mathbf{R}_e^b(r, p, y) \cdot$$

Observation-Equation for Sensor j on Platform i

$$\cdot [\ddot{\mathbf{x}}(t)^e - \mathbf{g}^e(\mathbf{x}) + 2 \cdot \boldsymbol{\Omega}_{ie}^e \cdot \dot{\mathbf{x}}(t)^e + \boldsymbol{\Omega}_{ie}^e \cdot \boldsymbol{\Omega}_{ie}^e \cdot \mathbf{x}(t)^e]_{S_{i,j}}$$



$$\mathbf{r}_{S_{i,j}}^{P_i} = \begin{bmatrix} \cos \delta \cdot \cos \alpha \\ \cos \delta \cdot \sin \alpha \\ \sin \delta \end{bmatrix}^{i,j}$$

Orientation j -th Sensor (s) on i -th Platform (p)

$$y = \left[x^e \ y^e \ z^e \mid \dot{x}^e \ \dot{y}^e \ \dot{z}^e \mid r^e \ p^e \ y^e \mid \ddot{x}^e \ \ddot{y}^e \ \ddot{z}^e \mid \omega_{eb,x}^b \ \omega_{eb,y}^b \ \omega_{eb,z}^b \mid \mathbf{s} \right]^T$$

$$(1) \ a_{s_{i,j}} = \mathbf{r}_{s_{ij}}^{p_i} \cdot \mathbf{R}_b^{p,i} \cdot \mathbf{R}_e^b(r, p, y)$$

Observation-Equation for Sensor j on Platform i

„l(i,j)“

$$\cdot [\ddot{\mathbf{x}}(t)^e - \mathbf{g}^e(\mathbf{x}) + 2 \cdot \boldsymbol{\Omega}_{ie}^e \cdot \dot{\mathbf{x}}(t)^e + \boldsymbol{\Omega}_{ie}^e \cdot \boldsymbol{\Omega}_{ie}^e \cdot \mathbf{x}(t)^e]_{s_{i,j}}$$

Referencing : Platform p(i) on Body (b) and Sensor s(i,j) on Platform (i) – „Leverarms“

$$(2) \ \mathbf{x}_{s_{i,j}}^e = \mathbf{x}_b^e + \mathbf{R}_b^e(r, p, y) \cdot \left[\mathbf{t}_{b,p_i}^b + \mathbf{R}_{p_i}^b \cdot \mathbf{t}_{p_i,s_{i,j}}^{p_i} \right]$$

Classical Kalman-Filtering as Least Squares or (robust) M-estimation In caes of Gauß densities

Procedure of Kalmanfiltering:

- 1.) Prediction of \mathbf{x} by the system equations (1) reading:

$$\mathbf{x}(i)_{i-1} = \mathbf{T}(\mathbf{x}(i-1), \mathbf{s}) \quad (3)$$

- 2.) Covariance matrix of the prediction:

$$\partial \mathbf{x}(i)_{i-1} = \underbrace{\begin{bmatrix} \dots & \frac{\partial \mathbf{T}_k}{\partial x_j} & \dots \end{bmatrix}}_{\bar{\mathbf{T}}} \cdot \partial \mathbf{x}(i-1) + \underbrace{\begin{bmatrix} \dots & \frac{\partial \mathbf{T}_k}{\partial s_l} & \dots \end{bmatrix}}_{\bar{\mathbf{S}}} \cdot \partial \mathbf{s} \quad (4)$$

$$\Rightarrow \mathbf{C}_{\mathbf{x}(i)_{i-1}} = \bar{\mathbf{T}} \cdot \mathbf{C}_{\mathbf{x}(i-1)} \cdot \bar{\mathbf{T}}^T + \bar{\mathbf{S}} \cdot \mathbf{C}_{\mathbf{ss}} \cdot \bar{\mathbf{S}}^T \quad (5)$$

- 3.) Filtering (Least Squares Adjustment, L2-Norm):
(Approximate state vector, filtering step i : \mathbf{x}_0)

$$\mathbf{x}(i)_{i-1} + \mathbf{v}_x = \mathbf{I} \cdot d\hat{\mathbf{x}} + \mathbf{x}_0 ; \quad \mathbf{C}_{\mathbf{x}(i)_{i-1}} \quad (6)$$

$$\mathbf{l}(i) + \mathbf{v}_l = \mathbf{A}(\mathbf{x}_0) \cdot d\hat{\mathbf{x}} + \mathbf{l}(\mathbf{x}_0) ; \quad \mathbf{C}_l \quad (7)$$

$$d\hat{\mathbf{x}} = \left[\mathbf{C}_{\mathbf{x}(i)_{i-1}}^{-1} + \mathbf{A}^T \mathbf{C}_l^{-1} \mathbf{A} \right]^{-1} \cdot \left[\mathbf{A}^T \mathbf{C}_l^{-1} \cdot (\mathbf{l} - \mathbf{l}(\mathbf{x}_0)) + \mathbf{C}_{\mathbf{x}(i)_{i-1}}^{-1} \cdot (\mathbf{x}(i)_{i-1} - \mathbf{x}_0) \right] \quad (8)$$

Definition of Kalman-Matrix : K

$$\mathbf{K} = [\mathbf{C}_{\mathbf{x}(i)_{i-1}}^{-1} + \mathbf{A}^T \mathbf{C}_1^{-1} \mathbf{A}]^{-1} \cdot \mathbf{A}^T \mathbf{C}_1^{-1} \quad (9)$$

$$d\hat{\mathbf{x}} = \mathbf{K} \cdot [(\mathbf{I} - \mathbf{I}(\mathbf{x}_0)) + \mathbf{C}_{\mathbf{x}(i)_{i-1}}^{-1} \cdot (\mathbf{x}(i)_{i-1} - \mathbf{x}_0)] \quad (10)$$

$$\hat{\mathbf{x}}(i) = \mathbf{x}_0 + \mathbf{K} \cdot [(\mathbf{I} - \mathbf{I}(\mathbf{x}_0)) + \mathbf{C}_{\mathbf{x}(i)_{i-1}}^{-1} \cdot (\mathbf{x}(i)_{i-1} - \mathbf{x}_0)] \quad (11)$$

Special choice: $\mathbf{x}_0 =: \mathbf{x}(i)_{i-1}$

$$\hat{\mathbf{x}}(i) = \mathbf{x}(i)_{i-1} + \mathbf{K} \cdot [(\mathbf{I} - \mathbf{I}(\mathbf{x}(i)_{i-1}))] \quad (12)$$

$$\mathbf{C}_{\hat{\mathbf{x}}(i)} = [\mathbf{C}_{\mathbf{x}(i)_{i-1}}^{-1} + \mathbf{A}^T \mathbf{C}_1^{-1} \mathbf{A}]^{-1} = [\mathbf{I} - \mathbf{K} \cdot \mathbf{A}] \cdot \mathbf{C}_{\mathbf{x}(i)_{i-1}} \quad (13)$$

- From (13) it follows, that the Kalman Filtering is more precise than the prediction.
- This does however not mean, that the Kalman-Filtering of the state vector \mathbf{x} in step i is more precise than the filtering before in step $(i-1)$. This question depends on the the influences, both of the law of error propagation concerning the prediction i from $(i-1)$, namely by (5) as well as by the system design \mathbf{A} , the control parameters \mathbf{s} and the system noise \mathbf{C}_{ss} .

Ground-Robots Particle Filter and SLAM



More flexible than Kalman-Filter: Particle-Filter

General Concept recursive Bayes-Estimation at time t, 1. Order Markov

$$\underbrace{p(\mathbf{y}_t | \mathbf{y}_0, \mathbf{l}_{0:t}, \mathbf{u}_{0:t})}_{\text{A-posteriori Density}} = \eta \cdot \underbrace{p(\mathbf{l}_t | \mathbf{y}_t)}_{\text{Sensor Measurements Density}} \cdot \int_{-\infty}^{+\infty} \underbrace{p(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{u}_t)}_{\text{Prediction-Density } \mathbf{t}_t} \cdot \underbrace{p(\mathbf{y}_{t-1} | \mathbf{l}_{0:t-1}, \mathbf{u}_{0:t-1})}_{\text{A-priori Density}} \cdot d\mathbf{y}_{t-1}$$

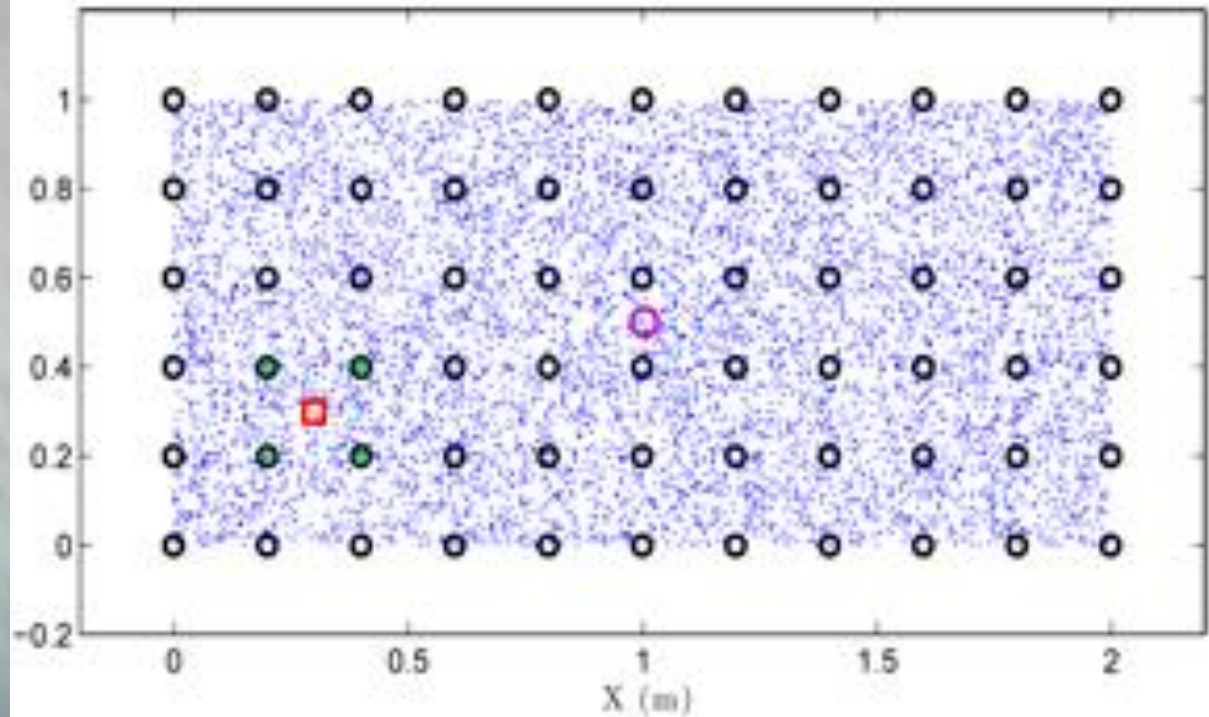
Chapman-Kolmogorov-Equation for Prediction of State from t-1 to t

$$p(\mathbf{y}_t | \mathbf{y}_0, \mathbf{l}_{0:t-1}, \mathbf{l}_t, \mathbf{u}_{0:t-1}, \mathbf{u}_t, \mathbf{y}_{t-1}) = \int_{-\infty}^{+\infty} p(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{u}_t) \cdot p(\mathbf{y}_{t-1} | \mathbf{l}_{0:t-1}, \mathbf{u}_{0:t-1}) \cdot d\mathbf{y}_{t-1}$$

Now: Approximation of the density of the preceding state (form the start state \mathbf{y}_0) by N „Particles“ via Dirac Delta Function $\delta(\mathbf{y}_{t-1} - \mathbf{y}_{t-1}^i)$

$$p(\mathbf{y}_{t-1} | \mathbf{l}_{0:t-1}, \mathbf{u}_{0:t-1}) = \sum_{i=1}^N w_{t-1}^i \cdot \delta(\mathbf{y}_{t-1} - \mathbf{y}_{t-1}^i) \quad \text{with} \quad \sum_{i=1}^N w_{t-1}^i = 1$$

Particle-Filter: Practical Use for the Localisation and Orientation indoors. E.g. Robot: Starting Situation



B-Building: $N = 30.000$ Particles

More flexible than Kalman-Filter: Particle-Filter

Informative Exploitation of the Prediction Model: $\mathbf{y}_t + \mathbf{s} = \mathbf{T}(\mathbf{y}_{t-1}, \mathbf{u}_t)$

Density Funktion of the Prediction

$$f(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{u}_t) = f_{\mathbf{s}}(\mathbf{s} = \mathbf{y}_t - \mathbf{T}(\mathbf{y}_{t-1}))$$

Use in Chapman-Kolmogorov Equation

$$p(\mathbf{y}_t | \mathbf{l}_{0:t-1}, \mathbf{l}_t, \mathbf{u}_{0:t-1}, \mathbf{u}_t, \mathbf{y}_{t-1}) = p(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{u}_t)$$

$$= \int_{-\infty}^{+\infty} \underbrace{f_{\mathbf{s}}(\mathbf{s} = \mathbf{y}_t - \mathbf{T}(\mathbf{y}_{t-1}, \mathbf{u}_t))}_{\text{Prediction Model}} \cdot \underbrace{\left(\sum_{i=1}^N w_{t-1}^i \cdot \delta(\mathbf{y}_{t-1} - \mathbf{y}_{k-1}^i) \right)}_{\text{Preceding State Estimation from former prediction and sensor measurements}} \cdot d\mathbf{y}_{k-1}$$

$$= \underbrace{\sum_{i=1}^N w_{t-1}^i \cdot f_{\mathbf{s}}(\mathbf{s} = \mathbf{y}_t - \mathbf{T}(\mathbf{y}_{t-1}^i, \mathbf{u}_t^i))}_{\text{Preceding State Estimation using Prediction-Model and Measurements}} = \left(\sum_{i=1}^N w_{t-1}^i \cdot \delta(\mathbf{y}_t - \mathbf{y}_t^i) \right)$$

More flexible than Kalman-Filter: Particle-Filter

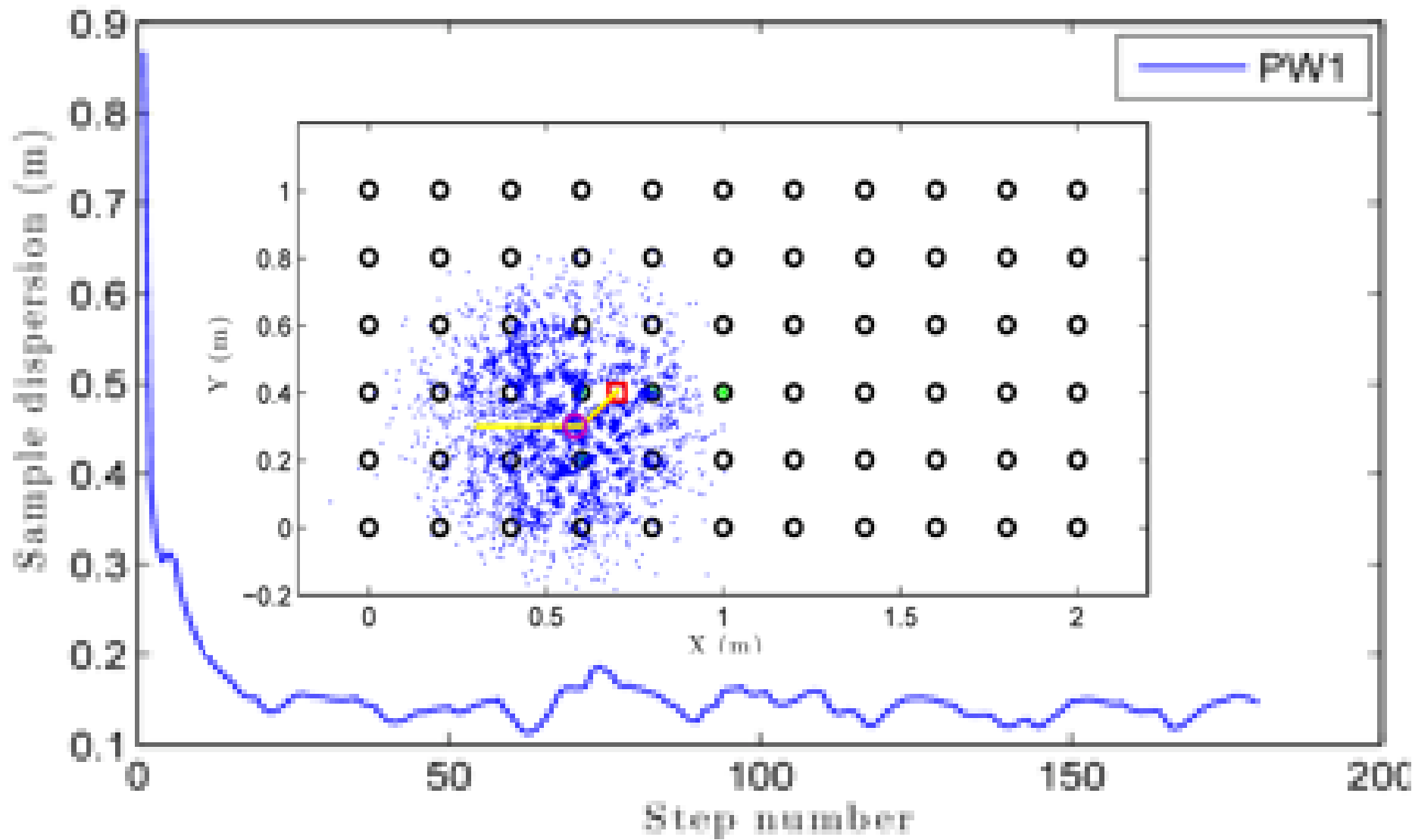
Now: Exploitation of the Information of the Observations

$$f(\mathbf{l}_t | \mathbf{y}_t) = f_e(\mathbf{l}_t - \mathbf{l}(\mathbf{y}_t))$$

$$\begin{aligned} f(\mathbf{y}_t | \mathbf{l}_{1:t}, \mathbf{u}_{1:t}) &= f_e(\mathbf{l}_t - \mathbf{l}(\mathbf{y}_t)) \cdot c \cdot \sum_{i=1}^N w_{t-1}^i \cdot \delta(\mathbf{y}_t - \mathbf{y}_t^i) \\ &= \sum_{i=1}^N \underbrace{c \cdot w_{t-1}^i \cdot f_e(\mathbf{l}_t - \mathbf{l}(\mathbf{y}_t^i))}_{w_t^i} \cdot \delta(\mathbf{y}_t - \mathbf{y}_t^i) \end{aligned}$$

Computation of new weights in step t with condition $\sum_{i=1}^N w_t^i = 1$

$$w_t^i = c \cdot w_{t-1}^i \cdot f_e(\mathbf{l}_t - \mathbf{l}(\mathbf{x}_t^i)) \quad \text{mit} \quad c = \frac{1}{\sum_{i=1}^N f_e(\mathbf{l}_t - \mathbf{l}(\mathbf{y}_t^i)) \cdot w_{t-1}^i}$$



SLAM ***Simultaneous*** ***Localization and*** ***Mapping***

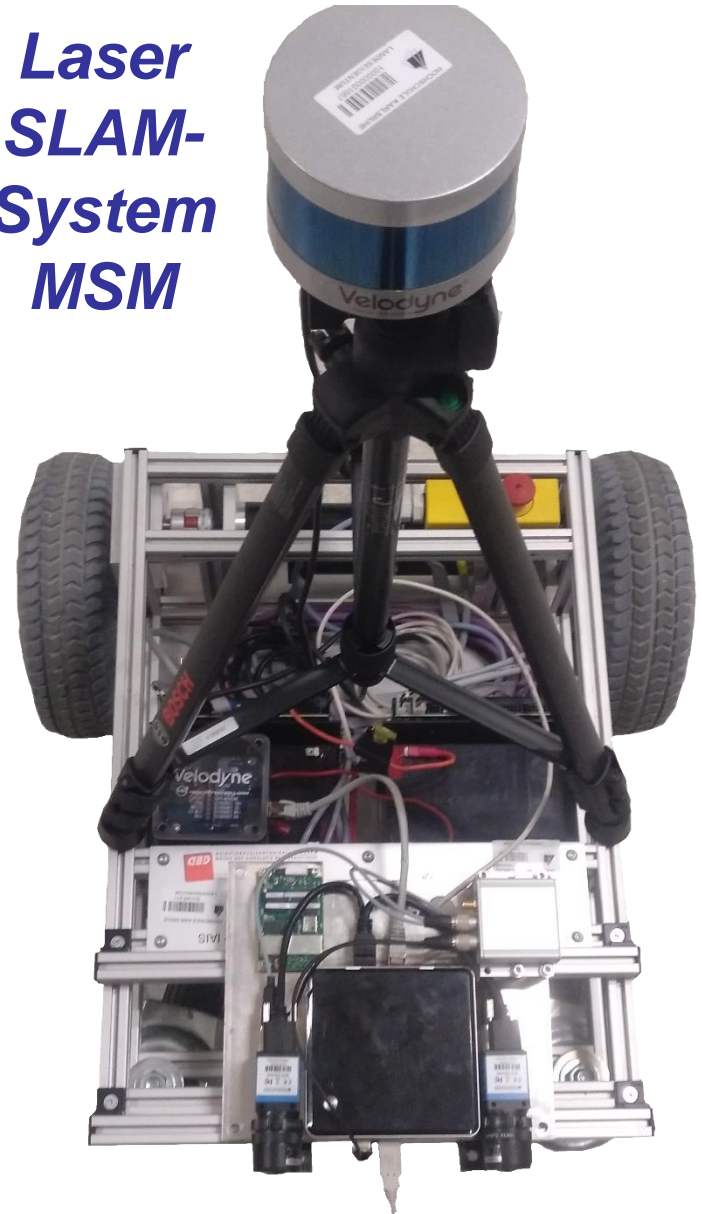


***Based on a Kalman-
or a Particle Filter
Navigation State
Estimation $y(t)$***

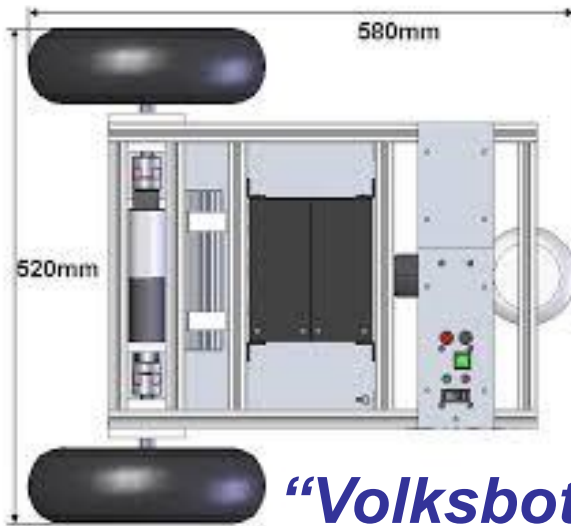
Ground-Robots – Particle-Filter and SLAM



**Laser
SLAM-
System
MSM**



**Odometry
Model
Improving
Prediction
Part**



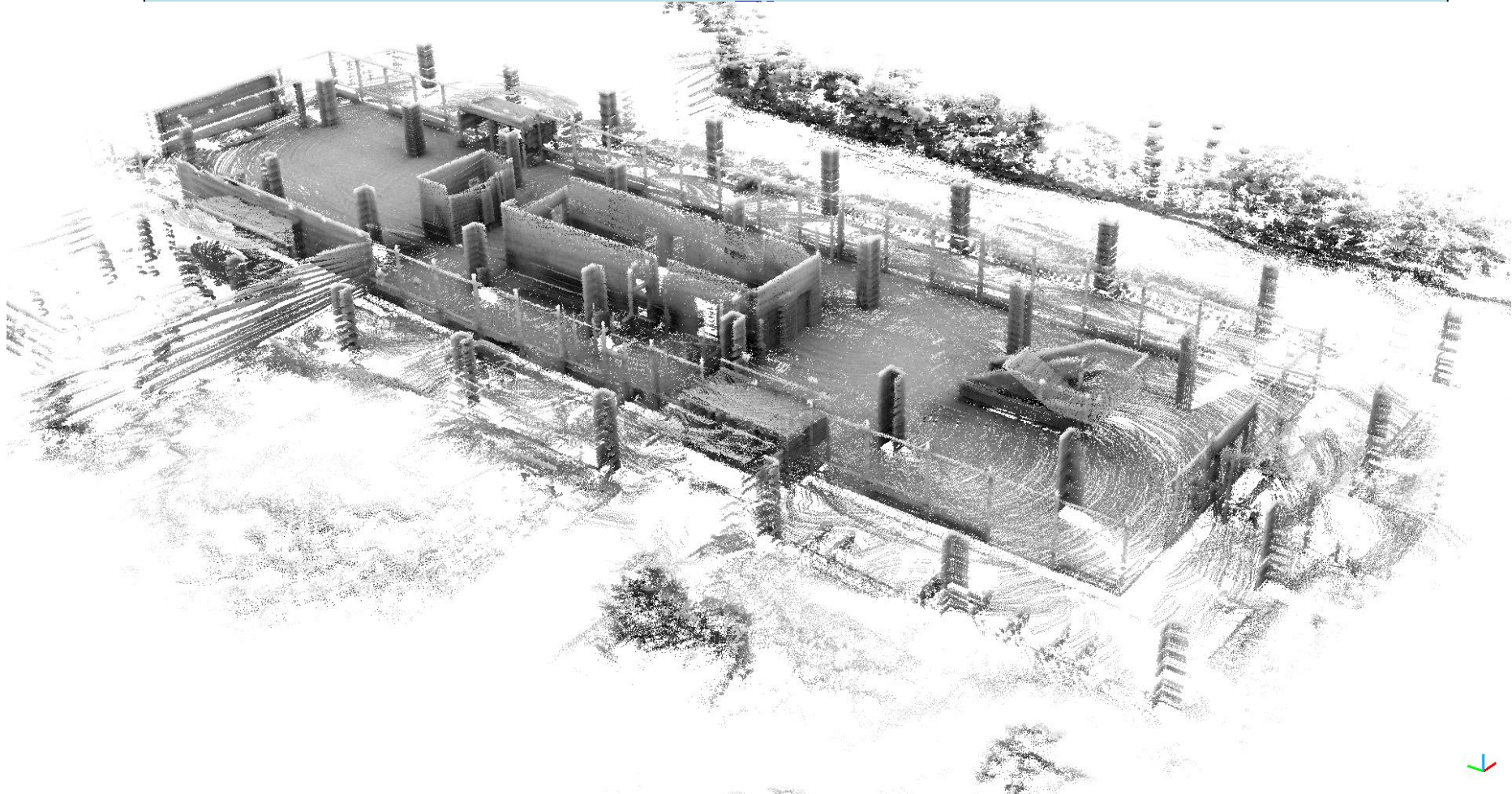
“Volksbot”

SLAM (Simult. Localization and Mapping) Development

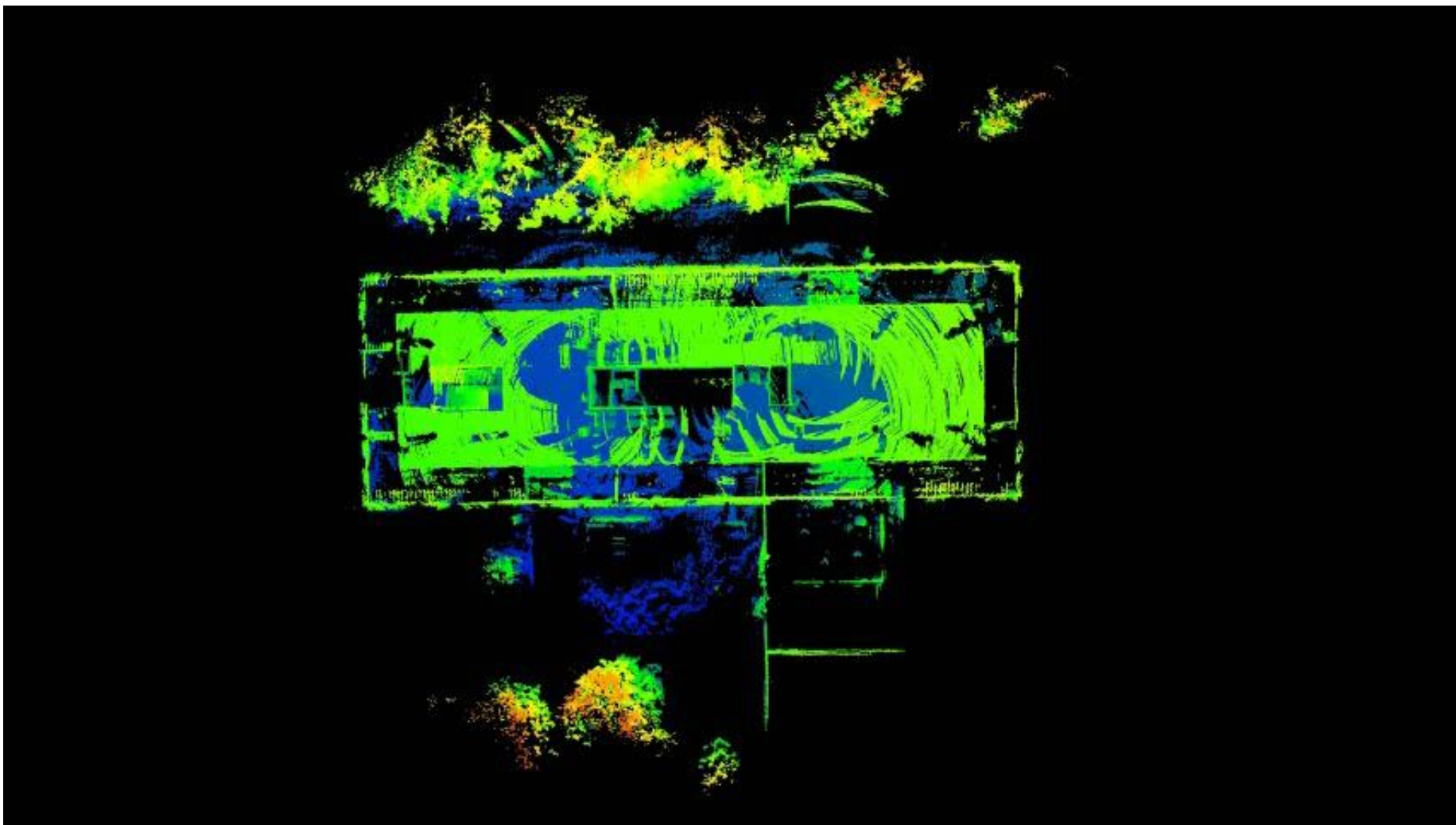
RaD “Multisensor Selfreferencing Localization & Mapping System (MSM)”

<http://www.navka.de/index.php/de/weitere-projekte/fue-projekte-produkte>

SLAM-based Mapping of B-Building HSKA, 2nd



SLAM-based Mapping of B-Building HSKA, 2nd Floor



UAV => UAS

Full Circuit

***Navigation
and Control***



Motion of Body



$$\vec{F}_T = \begin{pmatrix} 0 \\ 0 \\ -\sum_i (T_i) \end{pmatrix}$$

Total Motor Thrust F

$$\vec{M}_{T,i} = \vec{r}_i \times \vec{T}_i$$

Total Motor Torque M

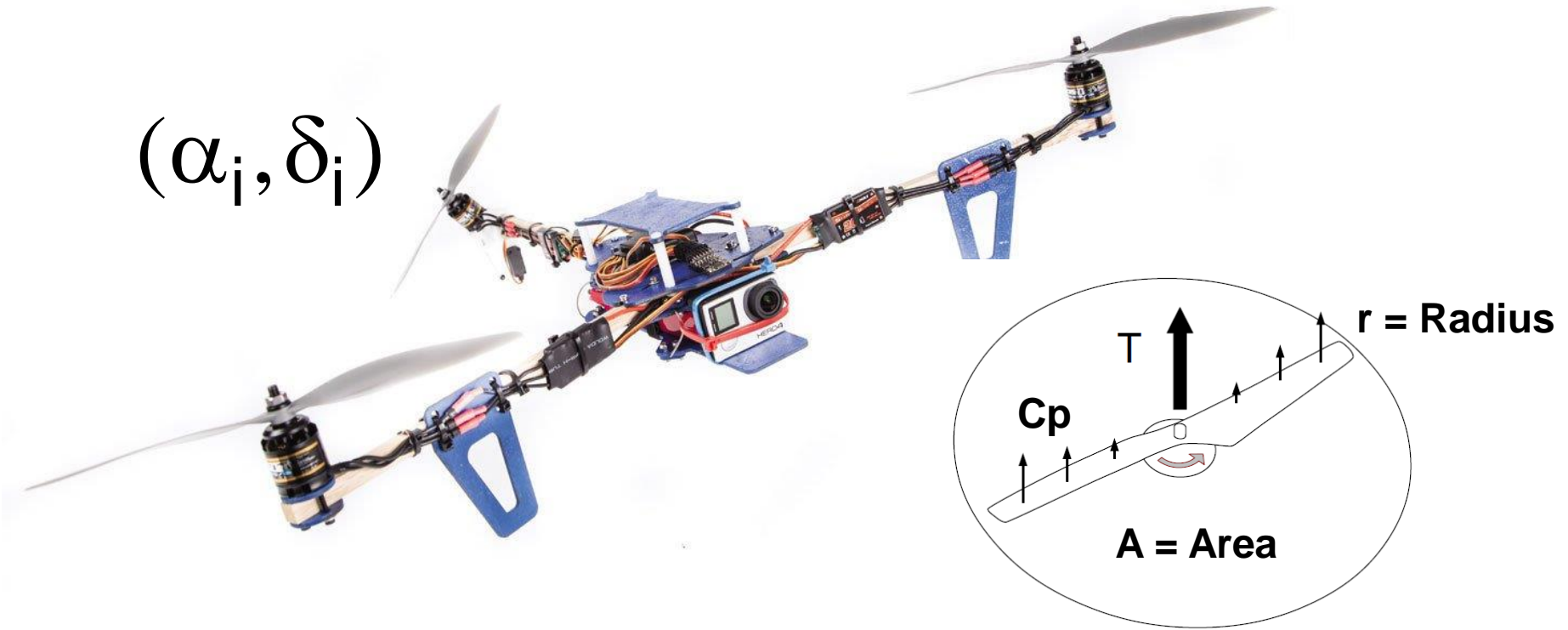
$$\vec{M}_T = \sum \vec{r}_i \times \vec{T}_i$$

$$= \begin{pmatrix} l \\ 0 \\ -h \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -T_V \end{pmatrix} + \begin{pmatrix} -l \\ 0 \\ -h \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -T_H \end{pmatrix} + \begin{pmatrix} 0 \\ -l \\ -h \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -T_L \end{pmatrix} + \begin{pmatrix} 0 \\ l \\ -h \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -T_R \end{pmatrix}$$

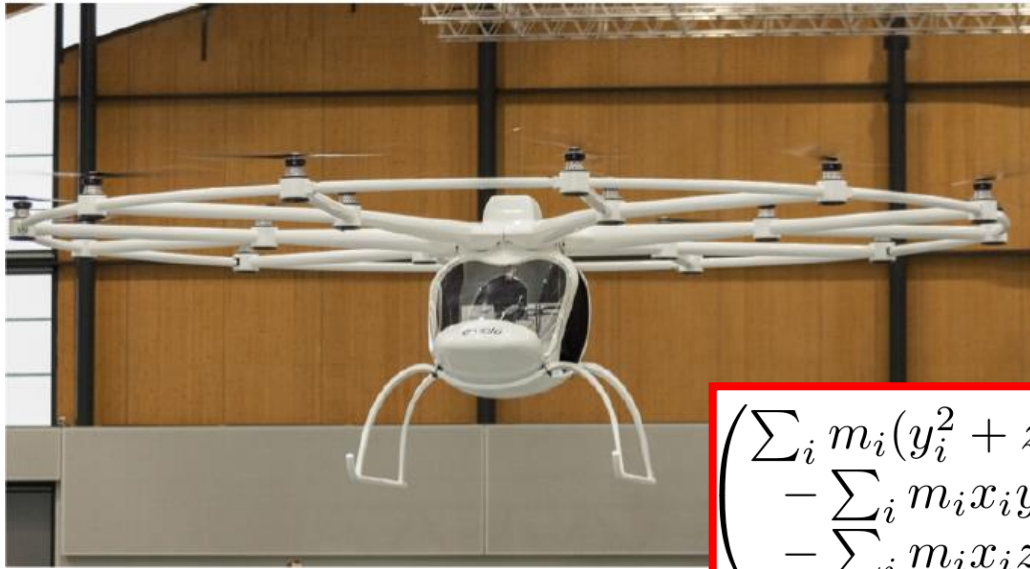
$$\vec{M}_T = \begin{bmatrix} I \cdot (T_L - T_R) \\ I \cdot (T_V - T_H) \\ 0 \end{bmatrix}$$

Flight Dynamics with arbitrary propeller orientation

(α_i, δ_i)



$$\mathbf{T}_i^b = \rho \cdot \pi \cdot c_i \cdot R_i^4 \omega_i^2 \cdot (\cos \alpha_i \cdot \sin \delta_i, \sin \alpha_i \cdot \sin \delta_i, -\cos \delta_i)^T$$



**Manned Volocopter VC 200
ZIM Project, Flight Control
by IAF / HSKA**

J =

$$\begin{pmatrix} \sum_i m_i (y_i^2 + z_i^2) & -\sum_i m_i x_i y_i & -\sum_i m_i x_i z_i \\ -\sum_i m_i x_i y_i & \sum_i m_i (x_i^2 + z_i^2) & -\sum_i m_i y_i z_i \\ -\sum_i m_i x_i z_i & -\sum_i m_i y_i z_i & \sum_i m_i (x_i^2 + y_i^2) \end{pmatrix}$$

Euler Equations

- **Discrete J Momentum of Inertia**
- **Dynamically changing J**

$$\mathbf{M}_{ges}^b = \sum_1^n \mathbf{M}_i^b + \mathbf{M}_{env}^b(d_i) = \boldsymbol{\omega}_{ib}^b \times (\mathbf{J} \cdot \boldsymbol{\omega}_{ib}^b) + \mathbf{J} \cdot \dot{\boldsymbol{\omega}}_{ib}^b$$

$$\mathbf{M}_i^b = \mathbf{r}_i \times \mathbf{T}_i^b$$

$$\omega_{P_i}, [i = 1, n]$$

- **General Propeller Design (l, h, r, Cp ,...)**



Manned Volocopter
VC 200

ZIM Project

Flight Control
by IAF / HSKA

Newton Equations

$$\omega_{P_i}, [i = 1, n]$$

$$\bar{\mathbf{F}}_{\text{total}} = \left[\frac{d(m \cdot \mathbf{v})}{dt} \right]_i$$

$$\bar{\mathbf{F}}_{\text{total}} = \bar{\mathbf{F}}^b + m \cdot \mathbf{R}_i^b \mathbf{g}^i = \left[\frac{d(m \cdot \mathbf{v})}{dt} \right]_b + \boldsymbol{\omega}_{ib}^b \times m \cdot \mathbf{v}_b = m \cdot \dot{\mathbf{v}}_b + \boldsymbol{\omega}_{ib}^b \times m \cdot \mathbf{v}_b$$

Flight Control Development Multicopter – n Prop.,no Symmetry

Control Deviation

(German: Regelabweichung)

$$\mathbf{e}(t) = \mathbf{y}(t)_{desired} - \mathbf{y}(t)_{Nav.State}$$

$$\begin{aligned} \left[\mathbf{F}_{ges}^b, \mathbf{M}_{ges}^b \right]^T &= \mathbf{F}(m, \mathbf{J}, \mathbf{g}^e, \mathbf{y}(t), \dot{\mathbf{y}}(t)) \\ &= \mathbf{F}_{PD}(m, \mathbf{J}, \mathbf{g}^e, \mathbf{y}(t), \mathbf{e}(t), \dot{\mathbf{e}}(t)) \end{aligned}$$

$$\left[\mathbf{F}_{ges}^b, \mathbf{M}_{ges}^b \right]^T =: \mathbf{u}'(t) = \mathbf{F}_{PD}(m, \mathbf{J}, \mathbf{g}^e, \mathbf{e}(t), \dot{\mathbf{e}}(t))$$

$$\begin{aligned} T &= (g + K_{z,D} (\dot{z}_d - \dot{z}) + K_{z,P} (z_d - z)) \frac{m}{C_\phi C_\theta}, \\ \tau_\phi &= \left(K_{\phi,D} (\dot{\phi}_d - \dot{\phi}) + K_{\phi,P} (\phi_d - \phi) \right) I_{xx}, \\ \tau_\theta &= \left(K_{\theta,D} (\dot{\theta}_d - \dot{\theta}) + K_{\theta,P} (\theta_d - \theta) \right) I_{yy}, \\ \tau_\psi &= \left(K_{\psi,D} (\dot{\psi}_d - \dot{\psi}) + K_{\psi,P} (\psi_d - \psi) \right) I_{zz}, \end{aligned}$$

$$\left. \begin{aligned} & \\ & \\ & \\ & \end{aligned} \right\} \mathbf{u}'(t) = \begin{bmatrix} T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \mathbf{u}(\mathbf{e}(t))$$

$\mathbf{u}'(t)$
Control Variable

1. Instance

Flight Control Development Multicopter – n Prop., no Symmetry

Control Deviation

(German: Regelabweichung)

$$\mathbf{e}(t) = \mathbf{y}(t)_{desired} - \mathbf{y}(t)_{Nav.State}$$

$$\begin{aligned} \left[\mathbf{F}_{ges}^b, \mathbf{M}_{ges}^b \right]^T &= \mathbf{F}(m, \mathbf{J}, \mathbf{g}^e, \mathbf{y}(t), \dot{\mathbf{y}}(t)) \\ &= \mathbf{F}_{PD}(m, \mathbf{J}, \mathbf{g}^e, \mathbf{y}(t), \mathbf{e}(t), \dot{\mathbf{e}}(t)) \end{aligned}$$

$$\left[\mathbf{F}_{ges}^b, \mathbf{M}_{ges}^b \right]^T =: \mathbf{u}'(t) = \mathbf{F}_{PD}(m, \mathbf{J}, \mathbf{g}^e, \mathbf{e}(t), \dot{\mathbf{e}}(t))$$

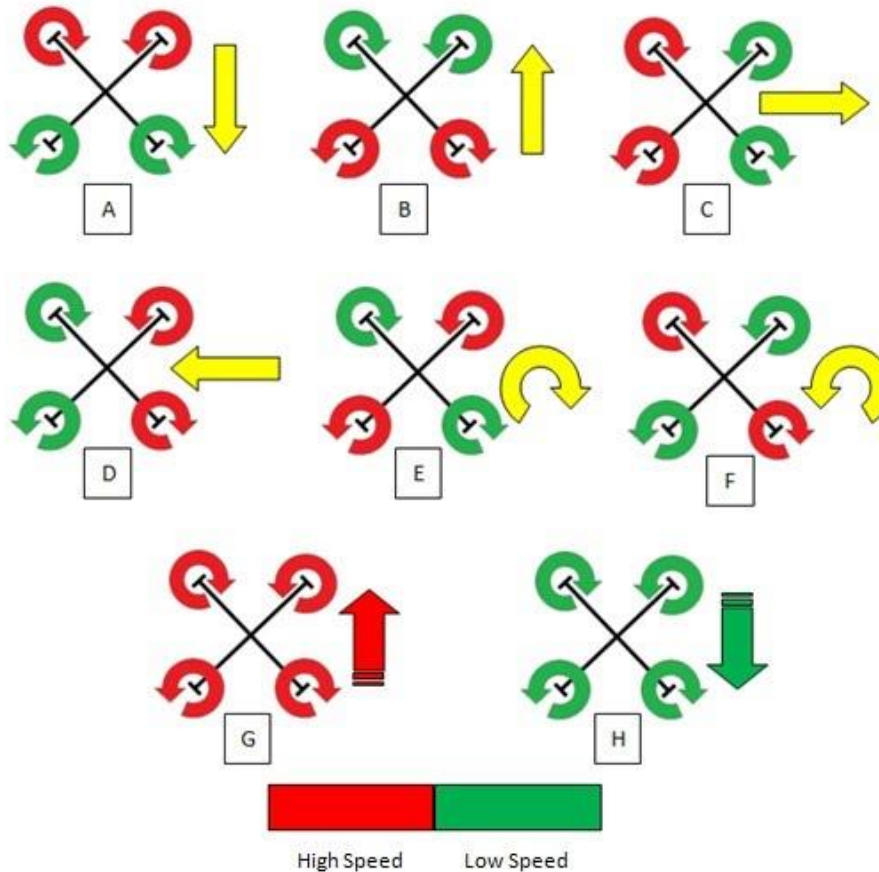
PD-Controller and PID-Controller, respect.

$$\mathbf{u}'(t) = \mathbf{K}_P \cdot \mathbf{e}(t) + \mathbf{K}_I \cdot \int_0^{\tau} \mathbf{e}(t) \cdot dt + \mathbf{K}_D \cdot \dot{\mathbf{e}}(t)$$

Flight Control Development Multicopter – n Prop.,no Symmetry

Control:

$\mathbf{e}(t) \Rightarrow \mathbf{u}'(t) \Rightarrow \mathbf{u}(t)$ Final Control Variables $\omega_{P_i}, [i = 1, n]$



$n=4$

$$\omega_1^2 = \frac{T}{4k} - \frac{\tau_\theta}{2kl} - \frac{\tau_\psi}{4b}$$

$$\omega_2^2 = \frac{T}{4k} - \frac{\tau_\phi}{2kl} + \frac{\tau_\psi}{4b}$$

$$\omega_3^2 = \frac{T}{4k} + \frac{\tau_\theta}{2kl} - \frac{\tau_\psi}{4b}$$

$$\omega_4^2 = \frac{T}{4k} + \frac{\tau_\phi}{2kl} + \frac{\tau_\psi}{4b}$$



**Control Deviation
(German: Regelabweichung)**

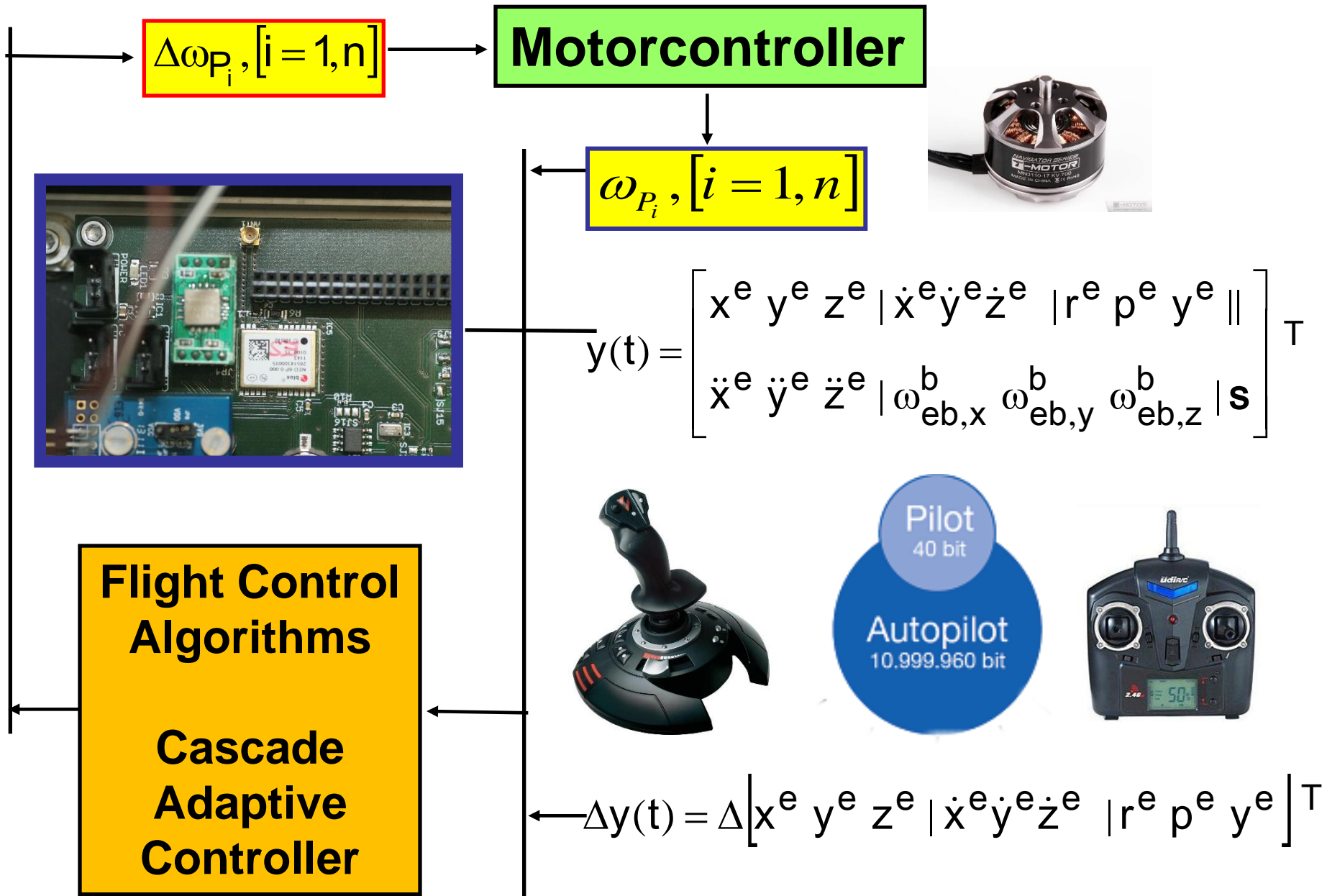
$$\mathbf{e}(t) = \mathbf{y}(t)_{\text{Soll}} - \mathbf{y}(t)_{\text{Ist}}$$

$$\mathbf{u}'_i(t) = \mathbf{K}_{p,i} \cdot \mathbf{e}_i(t) + \mathbf{K}_{I,i} \cdot \sum_0^{\tau} \mathbf{e}(t)dt + \mathbf{K}_{D,i} \cdot \frac{\mathbf{e}_i(t) - \mathbf{e}_i(t - \Delta t)}{\Delta t}$$

$$\Delta \mathbf{u}'(t) = [\Delta \mathbf{F}, \Delta \mathbf{M}]^T = \mathbf{K} \cdot [\Delta \omega_1, \dots, \Delta \omega_i, \dots, \Delta \omega_n]^T = \mathbf{K} \cdot \mathbf{u}(t)$$

$$\Delta \mathbf{u} = [\Delta \omega_1, \dots, \Delta \omega_i, \dots, \Delta \omega_n]^T = \mathbf{K}^{-1} \cdot [\Delta \mathbf{F}, \Delta \mathbf{M}]^T$$

NAVKA Flight Control Mathematical Model and Algorithms



Flight Control Developments - Multicopter n Propellers

ZIM-Project „E-Volocopter 2012-2015

6 Consortium Members (including IAF/HSKA)



Task of the Consortium Member IAF/ HS Karlsruhe
Development of the Flight-Control
Project-Leading: Prof. Dr.-Ing. Reiner Jäger

www.navka.de/index.php/de/weitere-projekte/abgeschlossene-projekte/e-volo-bemannte-multikopter

ZIM-Project „e-Volocopter 2012-2015“

Flight Control Developments - Multicopter n Propellers

IAF/HSKA



Flight Demonstration VC25 Ironbird (3m)
NAVKARINE “FC2”

Flight Control Developments Multicopter n Propellers

NAVKA-UAV and NAVKArine FC4 Flight Control Position Hold



IAF/HSKA
Flight Control
Developments
Multicopters
n Propellers

ZIM-Project
„e-Volocopter“
2012-2015

www.e-volo.com



NAVKArine-FC4
IAF/HSKA (right)

- Any Manned Volocopter
- Any UAV/UAS



e-Volopter, June-2017: AUTONOMOUS FLYING AIRTAXI in Dubai

www.e-volo.com

„Dubai beginnt
2017 weltweit
ersten Testbetrieb
autonomer
Lufttaxis mit dem
Volocopter“



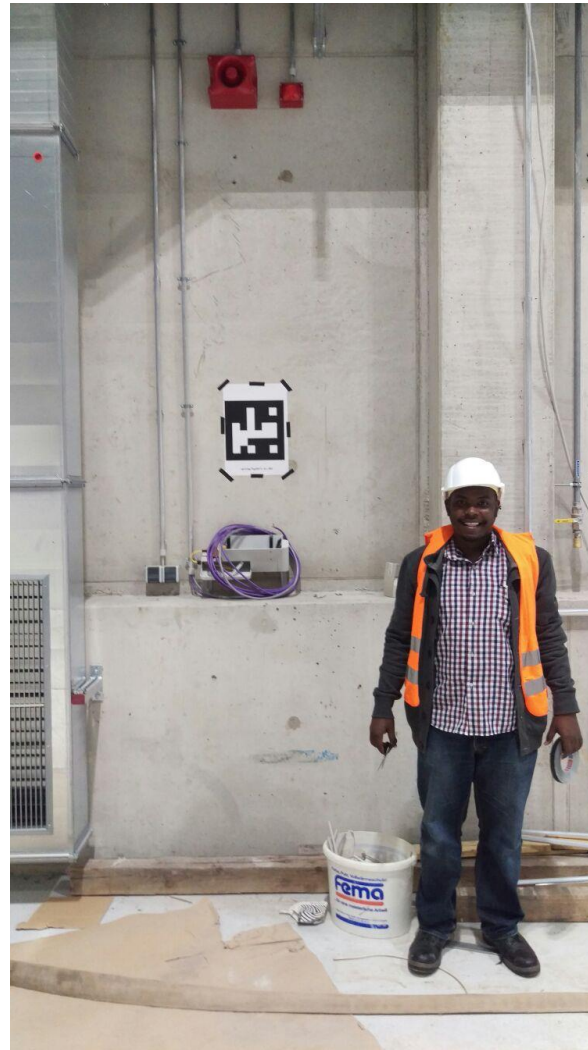
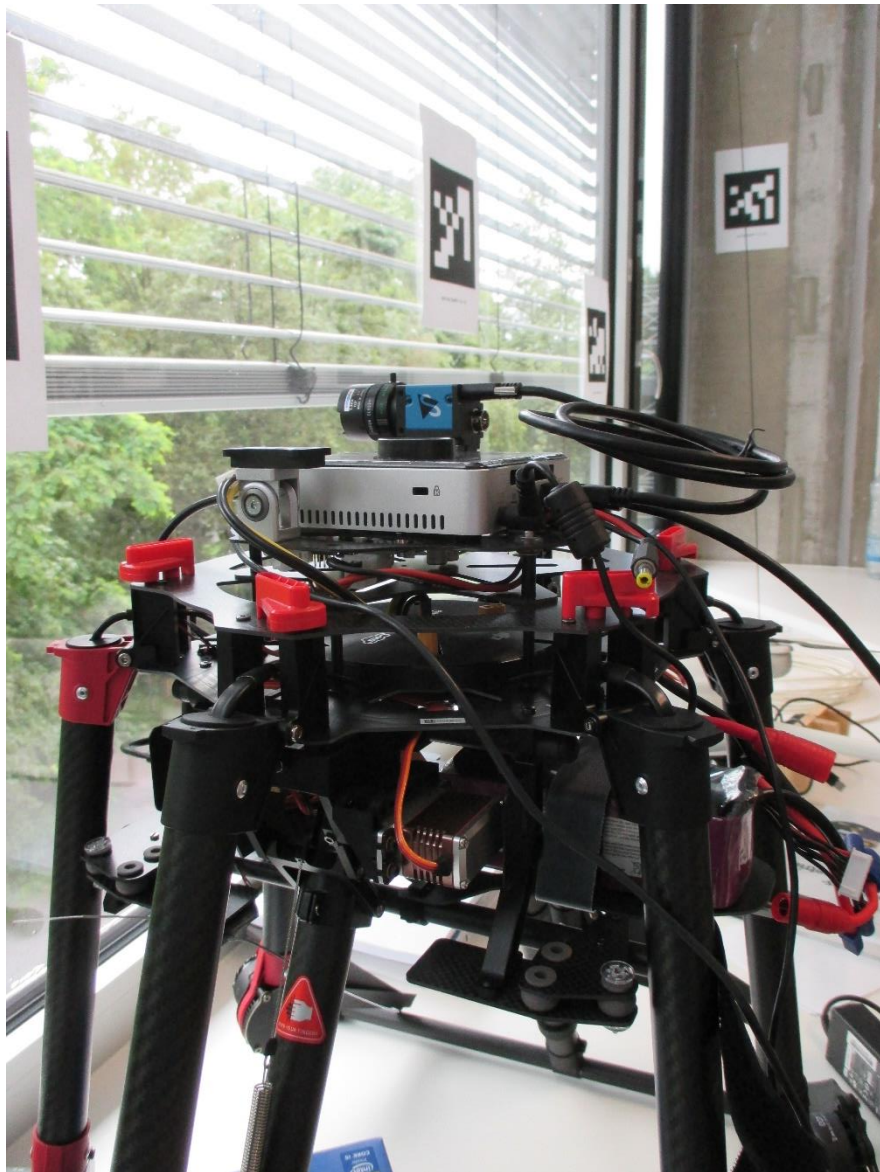
NAVKA Flight Control NAVKArine-FC4/FC5



- Environment friendly, silent: air taxis, individuals
- 3D mapping and geosensing
- Film industry
- Search and rescue of people
- Agriculture UAV
- Facility management & monitoring
- Wild life protection
- Transport UAV
- Fire Fighting air vehicles
- ABC sensing UAV for emergency event

NAVKA-Project – Daimler Indoor UAV

<http://www.navka.de/index.php/de/aktuelles/news>



**Marker Georeferencing
Daimler Crash Hall Sindelfingen**

NAVKA-Projekt – Multisensor Selfreferencing 3D-Mapping System (MMS)

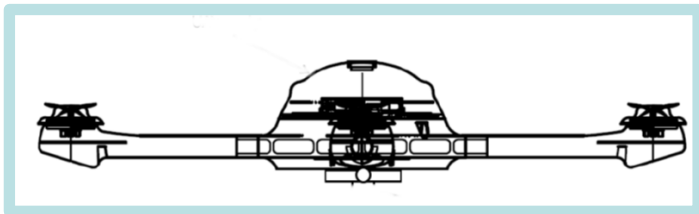
<http://www.navka.de/index.php/de/aktuelles/news>



<http://www.navka.de/index.php/de/ueberblick-msm>

NAVKA Seamless Out-/Indoor-Navigation-Concepts

Further Developments Multisensor Selfreferencing 3D-Mapping System (MSM)



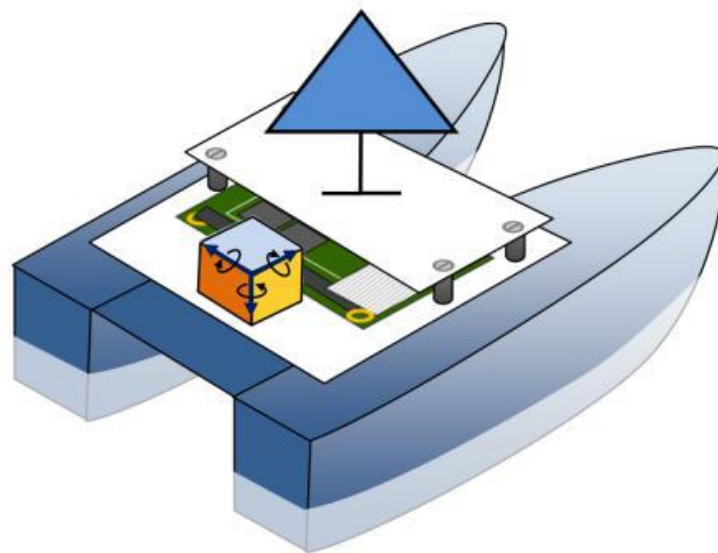
UAS NAVKArine-FC5 and 6



+
Light Weight
Laserscanner

LOD4 Building Models
NAVKA - SLAM-Algorithms

Scalable Multisensor Unmanned Maritim Vehicle (UMV)



Ground Vehicles

Multisensor Navigation



NAVKA RaD Project „Autonomous Out-/Indoor Driving“



UNTERNEHMEN PRODUKTE TI SALES SERVICE KOMMUNIKATION SHOP



GNSS: PPP-K
DGNSS: „Moving- Base“

<http://www.navka.de/index.php/de/aktuelles/news>

SELBSTANGETRIEBENE TRANSPORTER

SPEZIALTRANSPORTER



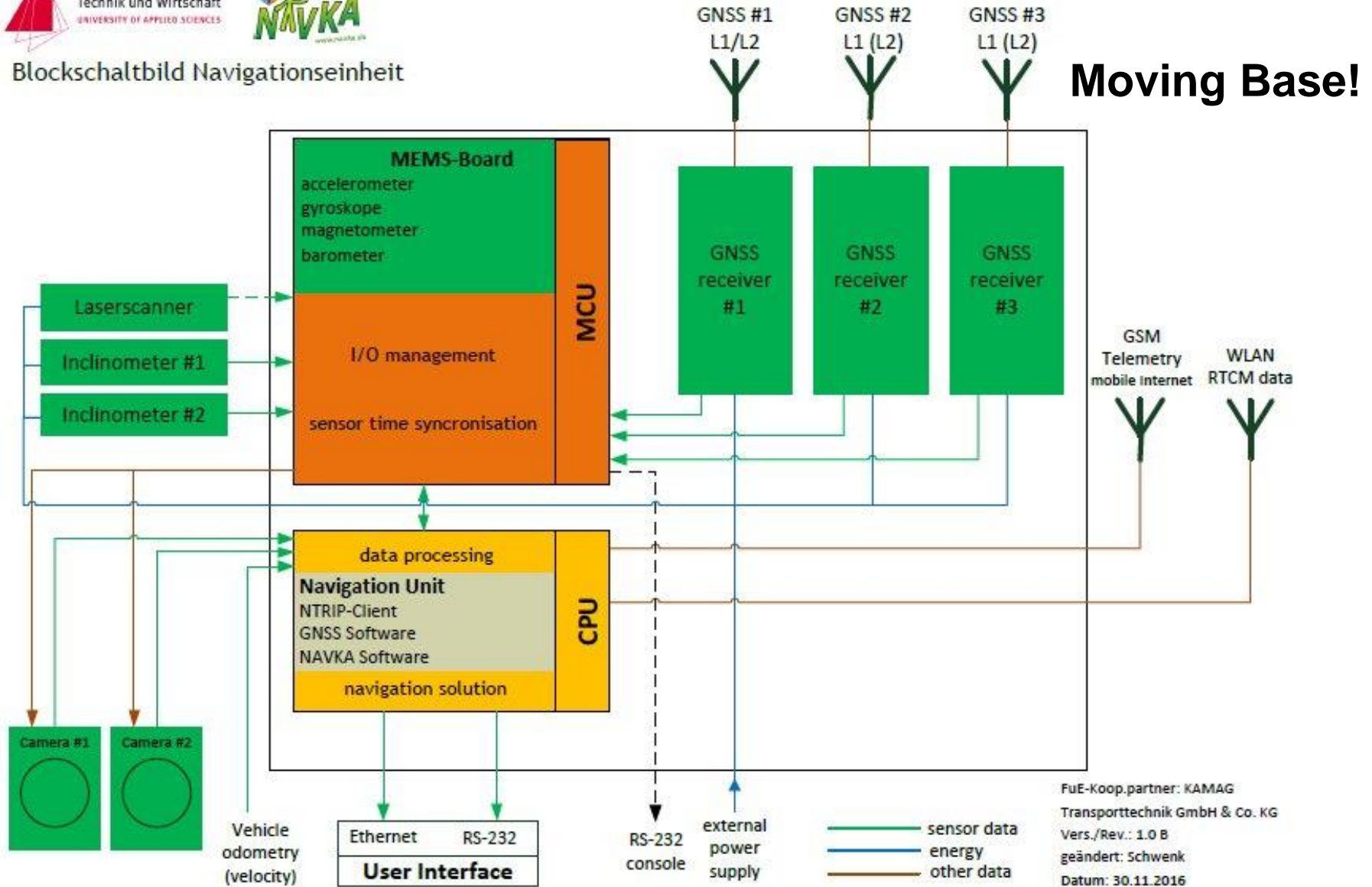
**Further
Sensor
Dynamic
Inclinometer**



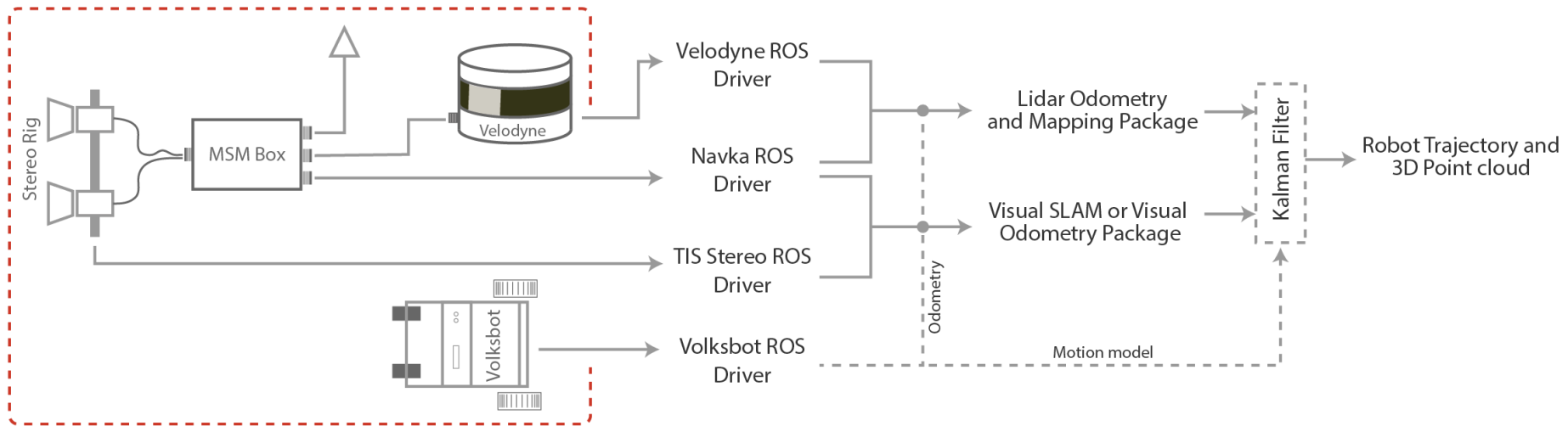
NAVKA RaD Project: KAMA Autonomous Out-/Indoor Driving“



Blockschaltbild Navigationseinheit

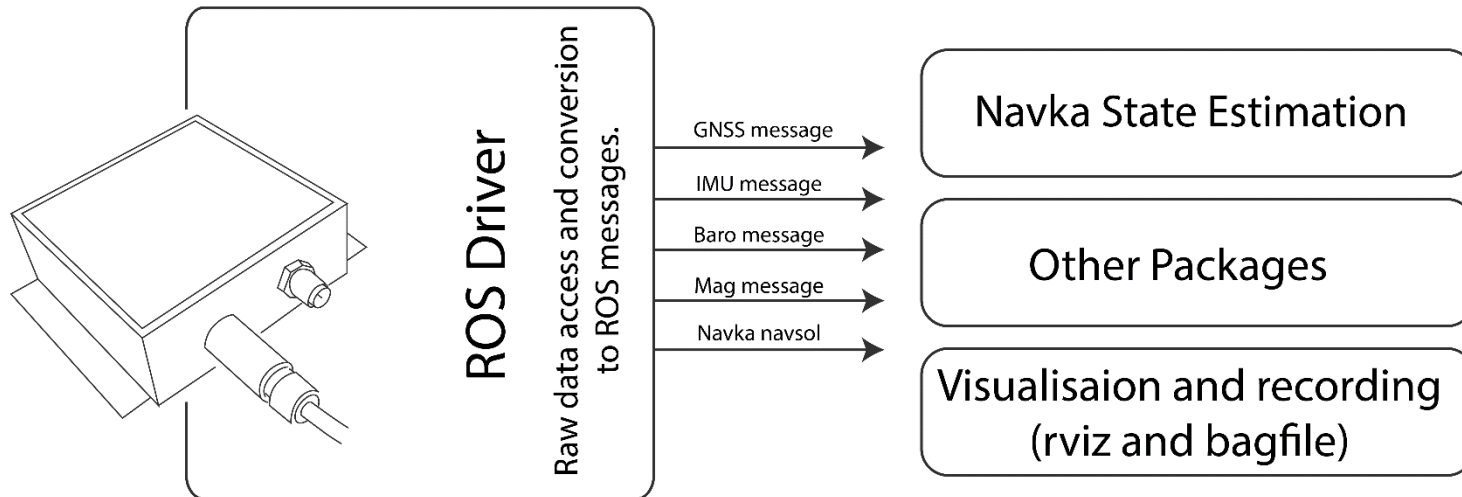


Partikelfilter => SLAM (Simultaneous Localization & Mapping)



FuE-Projekt Multisensorisches Selbstreferenzierendes Mapping System (MSM)

<http://www.navka.de/index.php/de/weitere-projekte/fue-projekte-produkte>



Examples for NAVKA Developments

**GNSS, MEMS, CAMERA
Out & Indoor**

**Real-Time Navigation
Kalman Filtering
of
Low-cost accelerometer,
gyroscope and GPS data
plus
Visual Odometry**

**GNSS & MEMS
Outdoor**





Further Information: www.navka.de

Book on Parameter-Estimation including Navigation 12/2017

<https://www.amazon.de/Klassische-robuste-Ausgleichungsverfahren-Ausbildung-Geoinformatikern/dp/3879076154>